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ABSTRACT

This volume contains abstracts of 33 research papers presented at the 53rd Annual Meeting of the National Council of Teachers of Mathematics. Most papers concerned instructional strategies; fourteen were concerned with sequencing of instruction and/or hierarchical analysis of learning. Four papers dealt with strategies for reviewing and reteaching. Six papers dealt with problems related to reading, verbalization and symbolic preference. Problem solving, deductive reasoning, inference patterns, and concept attainment were discussed in these contexts. Teacher education, attitudes toward mathematics, and characteristics of high-ability mathematics students were discussed in other papers. (SD)

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MATHEMATICS EDUCATION INFORMATION REPORT

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MATHEMATICS EDUCATION REPORTS

Denver, Colorado
April 23-26, 1975

RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

53rd ANNUAL MEETING

ERIC Information Analysis Center for
Science, Mathematics, and Environmental Education
400 Lincoln Tower
The Ohio State University
Columbus, Ohio 43210

PREFACE

The ERIC Information Analysis Center for Science, Mathematics, and Environmental Education has compiled abstracts of the research papers to be presented at the 53rd annual meeting of the National Council of Teachers of Mathematics. Selection of papers was made by Professor Len Pikaart of Ohio University and a committee of the AERA Special Interest Group for Research in Mathematics Education. Minor editing has been done by the ERIC staff to provide a general format for the papers. Many of the papers that are abstracted here will be published in journalf or be made available through the ERIC system. These will be announced in Research in Education and in the journal Investigations in Mathematics Education.

April, 1975

Jon L. Higgins
Associate Director for
Mathematics Education

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Minneapolis, Minnesota

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JUNE L. TRUDNAK, Bloomsburg State College
Bloomsburg, Pennsylvania

Reactor: LARS C. JANSSON, The University of Manitoba, Winnepeg
Manitoba, Canada

TOWARD A THEORY OF SEQUENCING: STUDY 3-5: THE SPECIFICATION AND VALIDATION OF A CURRICULUM HIERARCHY DESIGNED FOR USE IN TEACHING SELECTED PRINCIPLES AND STRATEGIES OF THE CLASS-REASONING ASPECT OF CRITICAL THINKING

Dr. Rocco Di Napoli
Three Village Central School District

Purpose

The development of a student's critical thinking ability is one of the implicit aims of education in general, and of mathematics education in particular. Currently, there exists much research evidence which indicates that many elementary, secondary, and college level students, including prospective elementary school teachers, are deficient in the rudimentary logical skills required to correctly judge the validity of certain types of simple, verbal, deductive arguments in class or conditional logic (Ennis and Paulus 1965; Gardiner 1966; Howell 1966; Paulus 1967; Miller 1968, 1969; O'Brien and Shapiro 1968, 1970; Roberge 1970, 1972; Morgan 1971; O'Brien, Shapiro, and Reali 1971; Jansson 1973; Shipman 1973). Apparently, there does not exist an empirically validated, hierarchical learning structure especially designed for the acquisition of this important aspect of critical thinking.

One approach in attempting to fill this void would be to first construct a theoretically sound hierarchical learning structure for the attainment of the desired competency over the desired content area, and then to test the validity of this hierarchy using precisely stated instructional hypotheses. This study represents a modest attempt at specifying and validating such a hierarchical learning structure--one designed especially for use in developing and improving the learner's ability to correctly judge the validity of simple, verbal, deductive arguments embodying the contrapositive and converse principles in class logic.

Procedure

The specification of the major learning hierarchies tested in the present study was accomplished via a task analysis of the terminal behavior listed below. This task analysis led to the identification of the set of two subordinate behaviors listed below.

Terminal Behavior

(V,J): the ability to correctly judge the validity of simple, verbal, deductive arguments in class logic.

Subordinate Behaviors

1. (V,S): the ability to correctly translate simple, verbal, deductive arguments in class logic into symbolic form.
2. (S,J): the ability to correctly judge the validity of simple, deductive arguments in class logic expressed in symbolic form.

Each of these behaviors was formulated as a precisely stated instructional objective for which carefully controlled computer-assisted instructional episodes were developed. The major aspect of the instruction developed for objective (V,S) was the use of simple rules and examples to explicate the concept of translating simple, verbal statements and arguments in class logic into symbolic form. The major aspect of the instruction developed for objective (S,J) involved the use of Venn diagrams to assist the learner in judging the validity of simple, deductive arguments in class logic expressed in symbolic form. A guided thinking information (GTI) instructional episode, designed to help students integrate the subordinate behaviors so as to promote their successful application to the performance of the terminal behavior, was also developed. These episodes were arranged into the following specific treatment sequences:

Hierarchy 1: (V,S) → (S,J) → GTI → (V,J)
Hierarchy 2: (V,S) → (S,J) → (V,J)
Hierarchy 3: (V,S) → (V,J)
Hierarchy 4: (S,J) → (V,S) → GTI → (V,J)
Hierarchy 5: (S,J) → (V,S) → (V,J)
Hierarchy 6: (S,J) → (V,J)

These treatment sequences were randomly administered, via an IBM 1500 Instructional System, to a sample of 194 prospective elementary school teachers enrolled in the elementary mathematics methods course (Math. Ed. 420) at The Pennsylvania State University, spring term, 1974.

Hypotheses

A carefully selected set of eight specific hypotheses were statistically tested, using the Binomial Probability Test, in order to decide upon the validity of each of the proposed learning hierarchies. In general, it was postulated that the set of behaviors listed in hierarchies 1 and 4 would constitute a sufficient set of subordinate behaviors for the acquisition of the terminal behavior, while those listed in hierarchies 2, 3, 5, and 6 would not.

Results

The results of the tests of hypotheses indicated that, subject to the limitations of this study, none of the six hierarchical learning structures was sufficient to enable 80% or more of the subjects who demonstrated mastery of the respective subordinate behaviors to acquire the terminal behavior. In addition, evidence was found to support the proposition that behaviors (V,S) and (S,J) are indeed subordinate to the terminal behavior (V,J).

Post hoc analyses showed that: a) the GTI instructional episode did function as predicted in hierarchy 1, but not in hierarchy 4 (perhaps because of the reverse order of acquisition of the subordinate behaviors), b) the order of attainment of the subordinate behaviors made no difference in terms of the successful attainment of the terminal behavior, c) hierarchy 1 and possibly 4 may be the more effective of the six hierarchies tested, d) any of the hierarchies 1, 2 or 4 and 5 may be used to obtain significant improvement in the terminal task for both principles tested, e) a much higher percentage of students could be expected to acquire the terminal behavior when both subordinate behaviors are applied to the terminal task than when only one or neither is applied, and f) the assumption that a competence is a competence regardless of the manner in which it was acquired may not be tenable.

Recommendations

In light of these results, the author recommends that this study be replicated with any or all of the following revisions: a) identify and include additional subordinate behaviors in the two major hierarchies, b) rework or expand the GTI instructional episode, c) rewrite the (V,S) and (S,J) episodes to include more practice, more repetition, and a longer training period, d) use more stringent conditions of mastery, e) use subjects at the secondary school level, and subjects possessing different levels of achievement at the same grade level, f) use other means of mediating instruction, g) use a different judgment procedure in the (S,J) episode, h) use other logical principles as the subject matter content, i) examine whether those students who are able to assess the validity of verbal arguments can also deduce valid conclusions from a given set of premises, and j) determine what effect, if any, positive or negative attitudes on the part of students toward CAI would have on their performance.

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TOWARD A THEORY OF SEQUENCING: STUDY 3-6:
AN EXPLORATION OF SPECIFIC TRANSFER PROPERTIES OF DIFFERENT INSTRUCTIONAL SEQUENCES DESIGNED FOR USE IN TEACHING SELECTED PRINCIPLES OF CLASS LOGIC

Peter J. Rizza Jr.
Control Data Corporation

Purpose

One set of factors to be examined when validating a learning hierarchy is the group of transfer properties generated by a specific learning sequence in the hierarchy. This investigation was designed to examine the specific transfer properties produced by instructional sequences developed for the teaching of selected principles of class logic. The purpose of this study was to assist in the improvement of the knowledge structures related to the perplexities of organizing curriculum and sequencing of instruction.

One of the principal objectives of education is the development of critical thinking in learners. Ennis (1962) described critical thinking as the correct assessing of statements. He further defines one stage of critical thinking as judging the necessity of a conclusion. This ability to judge simple deductive arguments is not naturally possessed or acquired through incidental learning, and therefore, without explicit instruction cannot be taken for granted (Ennis and Paulus, 1965; Gardiner, 1966; Paulus, 1967; Roberge and Paulus, 1971). It has been reported in these studies that in nearly all cases, without explicit instruction on specific principles, a vast majority of subjects failed to demonstrate mastery in the correct assessing of simple, verbal, deductive arguments in both class and conditional logic.

Problem

This investigation was designed to examine certain types of transfer demonstrated by preservice elementary school teachers as they interacted with an instructional module designed to enable them to judge the validity of simple, deductive arguments embodying selected principles of class logic. The two kinds of transfer inspected in the present study were first, the transfer of this process to other logical principles in the same content domain but not included in the instruction, and second, the transfer of this process to the same logical principles included in the instruction but expressed in a different content domain. The twenty hypotheses tested in this study were derived from a set of four general questions concerning the two types of transfer described above. The four general questions were designed to:

1) determine if, after achieving competency in judging verbal forms of simple deductive arguments in the class logic principles of contrapositive and converse in the concrete-familiar domain, the learner will transfer this competency to the principles of transitivity and inverse in the same domain.

2) determine if, after achieving competency in judging verbal forms of simple deductive arguments on the principles of contrapositive and converse in class logic in the concrete-familiar content mode of representation, the learner will transfer this competency to the same principles of contrapositive and converse in the suggestive content mode of representation.

3 determine if, after achieving competency in either of the two subtasks involving the principles of contrapositive and converse, the learner will transfer that competency to the principles of transitivity and inverse.

4) determine if, transfer at each subtask level to the principle of transitivity or inverse is necessary for transfer to the same principle at the terminal task level.

Table 1
Summary of Tests of Hypotheses

Hypothesis	Symbolic Form	Results
1.11	$-A(V,J) \wedge -A(V,J)^{3}p \nrightarrow A(V,J) \Longrightarrow A(V,J)^{3}p$	Supported
1.12	$-A(V,J) \wedge -A(V,J)^{3}p \nrightarrow -A(V,J) \Longrightarrow -A(V,J)^{3}p$	Not Supported
1.21	$-A(V,J) \wedge -A(V,J)^{4}p \nrightarrow A(V,J) \Longrightarrow A(V,J)^{4}p$	Supported
1.22	$-A(V,J) \wedge -A(V,J)^{4}p \nrightarrow -A(V,J) \Longrightarrow -A(V,J)^{4}p$	Supported
2.11	$-A(V,J) \wedge -A(V,J)^{1}s \nrightarrow A(V,J) \Longrightarrow A(V,J)^{1}s$	Supported
2.12	$-A(V,J) \wedge -A(V,J)^{1}s \nrightarrow -A(V,J) \Longrightarrow -A(V,J)^{1}s$	Supported
2.21	$-A(V,J) \wedge -A(V,J)^{2}s \nrightarrow A(V,J) \Longrightarrow A(V,J)^{2}s$	Supported
2.22	$-A(V,J) \wedge -A(V,J)^{2}s \nrightarrow -A(V,J) \Longrightarrow -A(V,J)^{2}s$	Supported
3.11	$-A(V,S) \wedge -A(V,S)^{3}p \nrightarrow A(V,S) \Longrightarrow A(V,S)^{3}p$	Supported
3.12	$-A(V,S) \wedge -A(V,S)^{3}p \nrightarrow -A(V,S) \Longrightarrow -A(V,S)^{3}p$	Not Tested

Table 1
(continued)
Summary of Tests of Hypotheses

Hypothesis	Symbolic Form	Results
3.21	$-A(V,S) \wedge -A(V,S)^{4}p \xrightarrow{A} A(V,S) \Longrightarrow A(V,S)^{4}p$	Supported
3.22	$-A(V,S) \wedge -A(V,S)^{4}p \xrightarrow{A} -A(V,S) \Longrightarrow -A(V,S)^{4}p$	Not Tested
3.31	$-A(S,J) \wedge -A(S,J)^{3}p \xrightarrow{A} A(S,J) \Longrightarrow A(S,J)^{3}p$	Not Supported
3.32	$-A(S,J) \wedge -A(S,J)^{3}p \xrightarrow{A} -A(S,J) \Longrightarrow -A(S,J)^{3}p$	Supported
3.41	$-A(S,J) \wedge -A(S,J)^{4}p \xrightarrow{A} A(S,J) \Longrightarrow A(S,J)^{4}p$	Supported
3.42	$-A(S,J) \wedge -A(S,J)^{4}p \xrightarrow{A} -A(S,J) \Longrightarrow -A(S,J)^{4}p$	Supported
4.11	$A(V,S)^{3}p \wedge A(S,J)^{3}p \wedge -A(V,J) \xrightarrow{A} A(V,J) \Longrightarrow A(V,J)^{3}p$	Supported
4.12	$-A(V,S)^{3}p \vee -A(S,J)^{3}p \wedge -A(V,J) \xrightarrow{A} A(V,J) \Longrightarrow -A(V,J)^{3}p$	Not Tested
4.21	$A(V,S)^{4}p \wedge A(S,J)^{4}p \quad -A(V,J) \xrightarrow{A} A(V,J) \Longrightarrow A(V,J)^{4}p$	Supported
4.22	$-A(V,S)^{4}p \vee -A(S,J)^{4}p \quad -A(V,J) \xrightarrow{A} A(V,J) \Longrightarrow -A(V,J)^{4}p$	Not Tested

Procedures

All 119 subjects were pre-service elementary school teachers enrolled in a mathematics methods course at the Pennsylvania State University. Those subjects who failed to show mastery of the terminal objective (JUDGING the Validity of Simple VERBAL deductive arguments in class logic (V,J)) on the pretest were considered subjects for this study. These subjects interacted with an instructional module mediated on an IBM 1500 System in a computer assisted instruction (CAI) setting. The module included instruction on two sub-task competencies:

1. (V,S) - translating VERBAL arguments into SYMBOLIC form
2. (S,J) - JUDGING the validity of arguments in SYMBOLIC form with the aid of Venn diagrams.

Pretests and posttests were administered at the sub-task level as well as tests for principle transfer. All students were administered a posttest to ascertain whether the terminal competence (V,J) was acquired and whether principle transfer or content-domain transfer had occurred.

Data Analysis

The statistical test used to determine if the pass/fail ratio of the subjects met the 80 percent criterion level was the Binomial Probability Test (Siegel, 1956), designed to examine populations consisting of only two classes. Since each of the subject groups formed in this investigation fell into either a "pass" or "fail" class on any given objective, this requirement was not violated. In all of the hypotheses tested $\alpha = .20$. The formula of the Binomial Test is as follows:

$$\sum_{i=0}^{x} \binom{N}{i} P^{N} Q^{N-x}$$

where:

N = the number of subjects meeting the initial condition of the hypothesis to be tested.

x = the subset of N subjects who have passed the stated condition of the hypothesis to be tested.

N-x = the subset of N subjects who have failed the stated condition of the hypothesis to be tested.

P = the predetermined proportion of subjects who should have passed the condition of the hypothesis to be tested in order to assume mastery (.80).

Q = 1-P (.20).

Results

Subject to the limitations of the generalizability imposed by the age of the students and the subject matter content specified, the following interpretations seem warranted:

1. The support of Hypotheses 1.11, 1.21, and 1.22 suggests that subjects who acquire the ability of judging the validity of simple deductive arguments embodying the principles of contraposition and conversion in the concrete-familiar content domain can transfer this competence to arguments embodying the principles of transitivity and inversion in the same content domain.

2. The support of Hypotheses 2.11, 2.12, 2.21, and 2.22 suggests that subjects who acquire this ability to judge the validity of simple, deductive arguments in class logic in the concrete-familiar content domain, embodying the principles of contraposition and conversion, can transfer this process to the same principles in the suggestive content domain.

3. The support of Hypotheses 3.11, and 3.21 suggests that subjects who acquire the process of *translating* simple, deductive arguments in class logic from *VERBAL* form to *SYMBOLIC* form can transfer that competence to other similar principles in class logic.

4. The support of Hypotheses 3.32, 3.41, and 3.42 suggests that, after subjects acquire the ability to *judge* the validity of simple, deductive arguments, containing only *one* universally quantified sentence, in *SYMBOLIC* form with the use of Venn diagrams, the subjects are capable of transferring this competence to arguments which also contain only *one* universally quantified sentence. However, non-support of Hypothesis 3.31 suggests that these subjects cannot transfer this process to arguments involving *two* universally quanitifed sentences.

5. The support of Hypotheses 4.11 and 4.21 suggests that those subjects who transferred the *terminal* competence (V,J) to other similar principles also transferred the *sub-task* competences (V,S) and (S,J) to other similar principles.

Recommendations

It has been demonstrated that transfer of training can be generated from instructional strategies that are designed to develop mastery of process competencies at a general level. As a result of the interaction with an instructional module so designed students were able to transfer competencies across both logic principles and content domain.

Since transfer is a goal of education, it is the recommendation of this study that those who develop instruction continue to search for the sort of competencies which permit the broadest range of transfer.

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TOWARD A THEORY OF SEQUENCING: STUDY 4-1: AN EXAMINATION OF THE EFFECTS OF A PARTICULAR CANONICAL TEACHING PROCEDURE ON CONCEPT ATTAINMENT AND GENERALIZATION IN MATHEMATICS

Harold J. Bailey
Bloomsburg State College

Purpose

The purposes of this study were to first examine the various effects of the SMSG canonical teaching procedure for permitting attainment of the set of instructional objectives, and then to examine the generalizability of the attainment of these instructional objectives over both the dimensional variable and the likelihood variable of the experimental model.

Procedure

A set of instructional objectives and generalization objectives were formulated for the study. Instructional objectives were those objectives for which explicit instruction included the appropriate use of all relevant attributes or the insignificance of the irrelevant attributes related to the objectives. In contrast, those objectives for which explicit instruction did not demonstrate the above relationships were classified as generalization objectives. All objectives considered in the study were a function of a set of variables which determined an experimental model - dimension, likelihood and embodiment. The embodiment variable remained constant throughout the study and consisted of the spinning of the pointer of a spinner. The dimensional variable referred to the number of times the pointer of the spinner was spun. The relationship of the size of the labeled regions on the spinner determined the likelihood variable.

The presentation of the concepts to the subjects occurred in one instructional setting through the use of instructional booklets. There were three forms of instructional booklets, one for each of three treatment groups employed in the investigation. The treatment groups were formed by the random assignment of 158 students enrolled in a Finite Mathematics course during the Spring Term, 1974, at Bloomsburg State College.

Hypotheses

Nine directional hypotheses were constructed and evaluated five times, once for each of the five hierarchical probability concepts considered in the study. The concepts, in the order in which they were disclosed to the learner were: outcome, sample space, event, mutually exclusive events and complementary events. All hypotheses were tested, at an $\alpha = .20$ level of significance, employing either the Binomial Test or the Fisher Exact Probability Test.

Results

In general, subject to the conditions of this investigation, the results were interpreted to indicate that the SMSG canonical teaching procedure was adequate for permitting attainment of the specified instructional objectives. That is, at least 80 percent of those students who failed to reach criterion on a particular instructional objective on a pretest and satisfied all other antecedent conditions, then reached criterion on the particular objective on a posttest. The data also suggested that explicit instruction on the irrelevant likelihood variable neither facilitated nor inhibited attainment of the related objective on a posttest. With respect to the dimensional variable, generalization from one-dimensional experiments to two-dimensional experiments did not occur for initial concepts in the hierarchy but did increase to a level of adequacy for a later concept. Generalizations from two-dimensional experiments to three-dimensional experiments were interpreted to occur for those concepts which did not require combinatorial analysis but did not occur for those concepts which required combinatorial analysis.

TOWARD A THEORY OF SEQUENCING: STUDY 4-2: THE RELATIVE EFFECTIVENESS OF FOUR CANONICAL TEACHING PROCEDURES FOR CONCEPT ATTAINMENT AND GENERALIZATION IN MATHEMATICS

June L. Trudnak
Bloomsburg State College

Purpose

This study was designed to investigate the relative effectiveness of four canonical teaching procedures for attainment and generalization of mathematical concepts. The four procedures were based upon a sequence of nine teaching acts initially proposed by the School Mathematics Study Group (SMSG) Panel on Research. Three of the teaching acts, re-defined in terms of Henderson's characterization, exemplification, and non-exemplification moves, were eliminated one at a time to form three of the instructional sequences, while the fourth procedure included all of the specified teaching acts. The total number of instructional moves for each concept was held constant by increasing the number of moves in those acts present in the sequence.

Procedure

The procedures were used to present five hierarchically related concepts from an introductory unit on elementary probability theory. The instruction presented experiments using spinners with two, four, or six different labels. Attainment of each concept was measured by a criterion performance test of four items using spinners similar, but not identical, to those used during the instruction.

Generalization of each concept was considered in both the primitive and mathematical sense as proposed by Dienes. Primitive generalization for each concept was investigated by asking the student to extend the class of spinners to other spinners with three or five labels. Mathematical generalization was studied by asking the student to use nonspinner embodiments with two, four, or six labels, which were elements of an isomorphic image of the spinners used for instruction.

Instructional objectives and related generalization objectives were written for each concept and were used for preparing the instructional materials for all four of the treatments. The instruction was given in the form of programmed booklets consisting of five parts, one for each concept. Instruction on a particular concept was preceded by a criterion performance, a primitive generalization, and a mathematical generalization pretest and followed by corresponding posttests. The subjects receiving the instruction were 124 students in College Algebra I at Bloomsburg State College who were randomly assigned to one of the four treatment groups.

Hypotheses

The experimenter hypothesized that with respect to concept attainment, primitive generalization, and mathematical generalization, the procedure with all nine teaching acts would be the most effective and that no significant differences would exist between those procedures with one of the teaching acts deleted. Each hypothesis was tested for each concept with $\alpha = .10$ using either the Fisher Exact Probability Test or the X^2 Test as dictated by the number of entries in the contingency table. In analyzing the data, passing a pretest eliminated the student from the test of the corresponding hypotheses. In addition, to be eligible for the tests of primitive and mathematical generalization hypotheses relative to a particular concept, a student must have first demonstrated attainment of that concept.

Results

The results indicated no significant differences in the treatment groups for concept attainment. However, significant differences were found with respect to generalization, but for only one concept. In this case, the complete sequence was found to be more effective than the sequence without exemplification moves for primitive generalization; and the procedure without nonexemplification moves was significantly more effective than those without initial characterization moves or without exemplification moves for primitive generalization; and the procedure without nonexemplification moves was significantly more effective than those without initial characterization moves or without exemplification moves for mathematical generalization.

Wednesday
3:30-4:30 p.m. RESEARCH REPORTING SECTION: NO. 22 Denver Room
Denver Hilton

Presider: CHARLES D'AUGUSTINE, Ohio University
Athens, Ohio

Reports: AN EVALUATION OF A PROGRAM OF CONTEMPORARY MATHEMATICS EDUCATION FOR PRE-SERVICE ELEMENTARY SCHOOL TEACHERS AT KENT STATE UNIVERSITY

MARIJANE WERNER, Kent State University
Kent, Ohio

AN EXPLORATION OF AN EVALUATION MODEL APPLIED TO THE INDIANA UNIVERSITY MATHEMATICS-METHODS PROGRAM INVOLVING THE ANALYSIS OF THREE DIMENSIONS OF TEACHING STYLE AND OTHER SELECTED VARIABLES AND THEIR RELATIONSHIP TO PUPIL ACHIEVEMENT

CAROL ANN DODD, Illinois State University
Normal, Illinois

THE MATHEMATICAL COMPETENCIES OF ELEMENTARY TEACHERS IN SELECTED UTAH SCHOOL DISTRICTS

DONALD D. CLARK, Utah State Department of Education
Midvale, Utah

ALLAN DAVIS, University of Utah
Salt Lake City, Utah

THE RELATIVE EFFECTIVENESS OF FOUR STRATEGIES FOR TEACHING ALGEBRAIC AND GEOMETRIC DISJUNCTIVE CONCEPTS AND FOR TEACHING INCLUSIVE AND EXCLUSIVE DISJUNCTIVE CONCEPTS

JOHN A. DOSSEY, Illinois State University
Normal, Illinois

A COMPARISON OF TWO LEVELS OF GUIDANCE AND TWO LEVELS OF ABSTRACTION IN PRESENTING NUMERATION CONCEPTS TO PROSPECTIVE ELEMENTARY TEACHERS

THOMAS P. CARPENTER, San Diego State University
San Diego, California

DOUGLAS B. MCLEOD, San Diego State University
San Diego, California

ROMUALDAS SKVARCLUS, Boston University
Boston, Massachusetts

AN EVALUATION OF A PROGRAM OF CONTEMPORARY MATHEMATICS EDUCATION FOR PRE-SERVICE ELEMENTARY SCHOOL TEACHERS AT KENT STATE UNIVERSITY

Marijane Werner
Kent State University

Purpose

The chief purpose of this study was to evaluate the undergraduate program of contemporary mathematics education for pre-service elementary school teachers (Ed 31125) at Kent State University at the end of the fifth year of implementation by determining cognitive and affective changes related to mathematics and the teaching of mathematics effected during the course. This evaluation was based upon empirical data collected during spring quarter, 1973 and statistically analyzed using three-way and four-way fixed effects analysis of variance and the Scheffe Contrast. In the analysis of variance statistics, if the F ratio is found to be significant at a specified level, the Scheffe Contrast is used to uncover the groups (in this case, cells) that contribute to the significant results. Direction for the research was provided by the general and specific behavioral objectives of the cognitive and affective domains and by the taxomony-type instruments designed to evaluate the objectives of both domains.

Another purpose of the study was to assess the cognitive and affective changes effected during the course between two different groups of college students, one group (S_{s1} with $n = 155$) from the Kent Campus and the other group (S_{s2} with $n = 19$) from the Stark Regional Campus where the implementation of the program differed from that on the Kent Campus. The two groups were compared at the beginning of the course and again at the end of the course over both the cognitive and affective domains. A total of 174 students took the pretest and posttest of both taxonomy-type instruments. Fifteen other students who completed the course during the spring quarter, 1973 were eliminated from this study because they were not present for the pretest of one or both instruments.

Results

After interpreting the results of the statistical analyses of the data, the following conclusions were drawn:

Hypothesis I - Statistically significant cognitive changes related to mathematics content and to the teaching of a program of contemporary mathematics to elementary-school children take place as a result of Ed 31125.

Hypothesis I was supported. Tables 1 and 3 of the report indicate that all variances were statistically significant ($p < .01$). The change score gains (Table 9) reveal that the subjects ($n = 174$) performed differently at different taxonomic levels and over course objectives of the cognitive domain. The significant mean gain of posttest over pretest scores supports hypothesis I. The Scheffe Contrast (Table 15) reveals that significant change ($p < .01$) occurred in objectives I and II at the knowledge, synthesis, and evaluation levels, objective III at the knowledge and evaluation levels, objective IV at the comprehension level, and objective V at the evaluation level.

Hypothesis II - Statistically significant changes in affect toward mathematics content and the teaching of a program of contemporary mathematics to elementary school children take place as a result of Ed 31125.

Hypothesis II was supported. Tables 5 and 7 of the report indicate that all variances over the taxonomic levels, course objectives, and mean gain of posttest over pretest scores were statistically significant ($p < .01$). The Scheffe Contrast reveals that significant change ($p < .01$) occurred in objective I at all levels of the affective domain, objective II at the first three levels, objective III at the last two levels, and objective IV at the third level of the affective domain.

Hypothesis III - Statistically significant cognitive differences related to mathematics content and to the teaching of a program of contemporary mathematics to elementary school children are found between $S_{s}1$ and $S_{s}2$ as a result of two different methods of implementing Ed 31125.

Hypothesis III was not supported. The difference in numbers between the two groups (Kent Campus with $n = 155$ and the Stark Regional Campus with $n = 19$) may account for failure of the data to support hypothesis III. The two groups did not perform very differently across taxonomic levels, and they performed very much the same across the five cognitive objectives. There was no significant difference between the two groups when total scores on the cognitive instrument were averaged over the pretest and over the posttest.

Hypothesis IV - Statistically significant affective differences related to mathematics content and to the teaching of a program of contemporary mathematics to elementary school children are found between S_{s1} and S_{s2} as a result of two different methods of implementing Ed 31125.

Hypothesis IV was supported. There was a statistically significant difference ($p < .01$) between the two groups when total scores on the affective instrument were averaged over the pretest and over the posttest. The affective change among students at the Stark Regional Campus was greater than the affective change of students at the Kent Campus. However, it should be noted that the good pretest affect of both groups became better by the end of the course.

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AN EXPLORATION OF AN EVALUATION MODEL APPLIED TO THE INDIANA UNIVERSITY MATHEMATICS-METHODS PROGRAM INVOLVING THE ANALYSIS OF THREE DIMENSIONS OF TEACHING STYLE AND OTHER SELECTED VARIABLES AND THEIR RELATIONSHIP TO PUPIL ACHIEVEMENT

Carol Ann Dodd
Illinois State University*

Purpose

The purpose of the study was to explore a technique for evaluating teacher education programs in terms of teaching competencies-- and, in particular, to explore that technique as applied to the Indiana University Mathematics-Methods Program. The evaluative procedures involved a process-product design in combination with a modification of Popham's performance test paradigm and aspects of the microteaching concept. Such a combination made possible the in-depth analysis of specific teaching behaviors as well as an assessment of pupil learning which could be related to the teaching.

Procedure

The sample of preservice teachers for the study consisted of 30 elementary education majors, 10 trainees randomly selected from the Mathematics-Methods Program and 10 randomly chosen from each of two contrasting programs providing mathematics preparation for prospective teachers at Indiana University (Encore and Regular Programs). The sample of 120 third grade pupils was randomly drawn from the total third grade population of two Bloomington schools. Five dependent variables were focused upon in the study: a preservice teacher's clarity in developing a mathematical idea (Clarity), the inquiry questioning techniques used in a mini-lesson (Questioning), the amount of encouragement given to constructive pupil involvement (Involvement), pupil achievement based on the mini-lesson (Pupil Score), and the mathematical background of preservice teachers relative to the topic of the mini-lesson (PST Test).

The preservice teachers were placed in a mini-teaching situation where pupil learning was the criterion of effectiveness. Each preservice teacher was given a list of instructional objectives upon which (s)he based an introductory lesson on equivalent fractions for four third grade pupils. The teaching of each PST was videotaped and subsequently analyzed by trained observers for Clarity, Questioning, and Involvement behaviors.

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*The research was conducted while the investigator was a Project associate of the Indiana University Mathematics-Methods Project.

Mean adjusted pupil posttest scores were used to assess pupil learning promoted by each preservice teacher. The analysis of the data tested differences in performance among the groups of preservice teachers on the dependent variables, and investigated the correlation between pupil achievement and the other four variables.

Results

Major findings were:

1. There were significant differences between groups on the multivariate composite of the five dependent variables. The MMP group performed highest, and those from the Regular Program performed lowest on all five variables.

2. On separate univariate tests, significant group differences were obtained for all variables except PST Test. Post hoc analyses indicated that the MMP group performed significantly higher than the Regular Program group on all four variables ($p < .05$), and significantly higher than the Encore Program group on Clarity, Involvement, and Pupil Score ($p < .10$). Preservice teachers from the Encore Program performed significantly better on the Pupil Score variable than did the Regular Program group ($p < .05$).

3. Significant positive correlations between Pupil Score and each of the variables Clarity, Questioning, and Involvement was obtained ($p < .01$).

Recommendations

1. The procedures used in the study should be considered a viable technique for evaluating segments of a teacher education program.

2. The three teacher variables Clarity, Questioning, and Involvement are strong correlates of pupil achievement.

3. The results of the study indicate that the Mathematics-Methods Program appears to have promoted competencies associated with the variables Clarity, Questioning, Involvement, and Pupil Score more effectively than the other two programs of the study.

4. There is an indication that regular, planned school experiences in conjunction with the mathematics preparation of preservice elementary teachers may have an impact on teacher competency to produce mathematical learning in children.

THE MATHEMATICAL COMPETENCIES OF ELEMENTARY TEACHERS IN SELECTED UTAH SCHOOL DISTRICTS

Donald D. Clark
Utah State Department of Education

Allan Davis
University of Utah

Purpose

This research examines the responses of elementary teachers on a 65 question test of mathematical comcepts which are taught in the first eight years of Utah public education. The mean scores of various groupings of teachers are compared...relative to locale of employing district, the sex of the teacher,...years of experience teaching mathematics...grade level taught...and the preservice training institution which recommended the teacher for certification.

The motivation for the study, which involved 333 teachers from districts of Utah, was to survey mathematics skills of a large sample of elementary teachers so as to provide data of use to teacher training institutions as well as to Utah school districts in developing their in-service programs and workshops for teachers.

Procedure

Twelve Utah school districts were included in the study. Teachers from selected schools in those districts were given a test generated by the study, the "Utah Elementary Teachers Mathematics Competency Evaluation Instrument." Teacher performance on this instrument provided the essential raw data for the study. The twelve districts included in the study were Jordan, Nebo, Box Elder, Murray, Salt Lake City, Ogden City, Park City, Millard, Beaver, Piute, South Sanpete, and Carbon. In each of these, 15% of the schools were identified and teachers in those schools were chosen as subjects to take the test instrument. Random processes were utilized in the selection of schools from given districts and in part in the selection of districts. Teachers from 25 schools were involved in the study and the target was to test all teachers of mathematics in these elementary schools. The coverage of teachers in the chosen schools was extremely good. Three hundred thirty-three teachers undertook the test.

The testing device, the "Utah Elementary Teachers Mathematics Competency Evaluation Instrument" is a modification and amalgamation of a 1965 test for elementary teachers developed by the School Mathematics Study Group and a 1969 test devised by Lita Schwartz et al. The various questions on the test are given in the study together with the number of correct responses, incorrect responses, and number of teachers not replying. The material being tested is divided into 19 general topics as primary targets. These are: Uniqueness of number, Exponents, Geometry, Set theory, Equivalence of fractions, Least common multiple, Division, Non-decimal systems, Percent, Computation with fractions, Computation with decimals, Averaging numbers, Verbal problems in mathematics, Inverse operations, Models for multiplication, Use of number line, Ordering of numbers, Logic, Solving equalities. Generally speaking, each topic is hit at in several different questions. Questions were of the multiple choice type.

Results

Out of a possible score of 65 on this test, the 164 teachers categorized as "primary teachers" attained an average score of 25.3 while the 163 "intermediate teachers" of the sample attained an average of 33.4 correct responses. Thus the "average" elementary school teacher tested achieved a score of about 29 correct responses on the 65 questions.

Conclusions drawn are the following:

1. There is no significant difference in the mathematical competencies of (tested) teachers when stratified into three groups by years of experience teaching elementary mathematics.

2. There is no significant difference in the mathematical competencies of teachers employed by the Wasatch front districts and those teachers employed in other districts in Utah.

3. There is a significant difference between the competency of elementary primary grade teachers and intermediate grade teachers. (As noted above, the mean score for primary teachers on the test instrument was 25.3 whereas it was 33.4 for the intermediate grade teachers.)

4. The elementary teachers in Utah demonstrated a similar level of proficiency on the instrument of this study as elementary teachers in other states have achieved on comparable tests.

5. Regarding teacher training institutions in Utah, "in general there were no differences in teachers from the different institutions."

6. In general, male and female elementary teachers achieved equally well on the instrument of this study.

7. To improve the total mathematical knowledge of the students in Utah's public schools the elementary teachers will have to increase their own mathematical competencies.

Recommendations

Dr. Clark regards the results of his study as indicating a state wide concern and calling for in-service courses in mathematical content to be established as a cooperative effort throughout the State. His point is that the elementary teachers in Utah need to understand the concepts in mathematics which are included in the curriculum of the first seven grade levels of the public school system. He argues that children of Utah deserve to be taught by mathematically competent teachers. Yet he finds very broad gaps in teacher knowledge of mathematics. It is slight solace that Utah elementary teachers seem to do about as well as elementary teachers elsewhere in the U.S. do on comparable mathematics tests. His point is that this level could be greatly improved for the benefit of Utah students.

Dr. Clark calls for improvements in the programs of the higher education institutions of the State in the mathematical training of elementary teachers and also for enlarged and expanded efforts for the training or retraining in mathematical content of in-service elementary school teachers of the State.

His careful study is a mine of interesting information and deserves the serious attention of all persons concerned with elementary instruction in Utah. It has already spawned a number of other research projects by various investigators and it is expected to have a profound effect on the Utah elementary school mathematics scene in the years just ahead.

THE RELATIVE EFFECTIVENESS OF FOUR STRATEGIES FOR TEACHING ALGEBRAIC AND GEOMETRIC DISJUNCTIVE CONCEPTS AND FOR TEACHING INCLUSIVE AND EXCLUSIVE DISJUNCTIVE CONCEPTS

John A. Dossey
Illinois State University

Purpose

The purpose of this study was to ascertain the role played by four different teaching strategies and exemplification approaches in promoting Ss' acquisition of a set of algebraic and geometric disjunctive concepts. In an earlier paper dealing with the relative effectiveness of the approaches, significant differences were found between the Ss' attainment of algebraic and geometric disjunctive concepts, as well as between their attainment of inclusive and exclusive disjunctive concepts (Dossey & Henderson, 1974). These findings, coupled with others, suggest that the subject matter and logical nature of a concept make a difference in the effectiveness of an instructional strategy for teaching concepts. The purpose of this study was to describe the role that the teaching strategies and exemplification approaches in the first study played in the Ss' attainment of the disjunctive concepts used.

The study was designed about a 4 X 2 X 2 factorial design. The first factor represented the four different teaching strategies used. They were molded to fit the model for concept teaching developed by Henderson (1970). The four strategies used were:

1. Characterization-Exemplification Strategy (CE). An instructional strategy consisting of four characterization moves followed by a set of six exemplification moves.

2. Characterization-Exemplification-Characterization Strategy (CEC). An instructional strategy consisting of two characterization moves, followed by six exemplification moves, followed by an additional set of two characterization moves.

3. Exemplification-Characterization-Exemplification Strategy (ECE). An instructional strategy consisting of three exemplification moves, followed by a set of four characterization moves, followed by an additional set of three exemplification moves.

4. Exemplification-Characterization Strategy (EC). An instructional strategy consisting of six exemplification moves followed by a set of four characterization moves.

These four strategies were chosen as a result of the analysis of several patterns of teachers' verbal patterns of behavior in the teaching of disjunctive concepts.

The second factor of interest was the ratio of example moves to nonexample moves within each of the sets of six exemplification moves contained within each of the strategies. The two levels of the exemplification approach factor considered were:

1. Example Approach (E). The six exemplification moves in each of the strategies in this combination of the factors were divided into four example moves and two nonexample moves.

2. Nonexample Approach (∿E). The six exemplification moves in each of the strategies in this combination of the factors were divided into two example moves and four nonexample moves.

Hence, the example approach had a 2 to 1 ratio of example moves to nonexample moves, while the nonexample approach had a 1 to 2 ratio of example moves to nonexample moves.

The third factor of experimental interest was that of intellectual ability. It was incorporated into the study to see if the above two factors might have differing effects on Ss of differing intellectual ability levels.

Procedure

The combination of the above factors into the 4 X 2 X 2 completely crossed factorial design resulted in a model with 16 cells. Twenty prospective elementary school teachers were classified according to intelligence on a general mental ability test and then randomly assigned to the cells of the model. They were each then provided with programmed instructional materials appropriate to their cell. The materials introduced the Ss to twelve disjunctive concepts which were balanced with respect to being algebraic and geometric as well as with respect to being inclusive or exclusive in logical nature. Following the completion of the units, the Ss took a 72 item test covering the twelve concepts. These test scores were used in the analysis of the effectiveness of the various approaches.

Results

The analysis of the data showed that the strategy factor was found to have significant differences among its levels in the case of the algebraic concepts and the inclusive disjunctive concepts. Since no hypotheses had been formulated prior to the analysis of variance concerning the direction of the differences, Scheffe's Test for Multiple Comparisons was applied to the treatment sums of the strategy factor levels to determine which strategies differed at the 0.05 level.

In the case of the algebraic concepts, the only difference judged significant by Scheffe's test was the one between the CEC and ECE strategies. A comparison of the means for each of the strategies showed that the CEC strategy was more efficacious than the ECE strategy in promoting the Ss' acquisition of algebraic disjunctive concepts.

In the case of the inclusive disjunctive concepts, the strategy differences judged significant by Scheffe's Test were between the CEC and ECE strategies and between the combination of groups taught by the CEC and EC strategies and those taught by the CE and ECE strategies. In both cases the group mentioned first was judged to be the better approach.

The exemplification approach was judged to have a significant effect on the Ss' learning in the case of the inclusive disjunctive concepts. The examination of the approach means indicated that the Example Approach was more efficacious than the Nonexample Approach in promoting student learning of inclusive disjunctive concepts.

In all of the cases, the groups from the higher intellectual ability cells did significantly better than those from the lower intellectual ability cells, as was expected.

In addition to the above findings, some unusual, but perhaps meaningful 3-way interactions were noted in the case of the geometric and the exclusive disjunctive concept cases. These findings, coupled with those mentioned earlier, suggest that the opening of a disjunctive concept teaching strategy with a set of characterization moves may be a very effective strategy. Further research is being conducted to follow up on this.

The strategy differences also indicate that the role and order of the moves in a disjunctive concept teaching strategy may have a great effect on the Ss' ability to infer what the defining characteristics of the concept may be. Here is where the role of the characterization moves is important.

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A COMPARISON OF TWO LEVELS OF GUIDANCE AND TWO LEVELS OF ABSTRACTION IN PRESENTING NUMERATION CONCEPTS TO PROSPECTIVE ELEMENTARY TEACHERS

Thomas P. Carpenter
San Diego State University

Douglas B. McLeod
San Diego State University

Romualdas Skvarcius
Boston University

Purpose

Research on discovery learning has often confused the effects of level of guidance with various levels of abstraction. These are distinct dimensions that can be varied independently. In fact, these dimensions may interact with one another.

The purpose of this study was to compare two levels of guidance (discovery and expository modes) and two levels of abstraction (use of multibase blocks and a symbolic presentation not using blocks) in presenting numeration concepts in bases other than ten.

Procedure

Sixty students from two sections of a mathematics class for elementary teachers were randomly assigned to one of four treatment groups: discovery with multibase blocks (DM), expository with multibase blocks (EM), discovery without using blocks (DN), and expository without using blocks (EN). Each student worked individually on a packet of printed materials designed for his treatment group. While the printed materials varied in level of guidance and level of abstraction, they covered the same content using the same sequence and the same problems, and they provided the same amount of practice.

Minimal guidance was provided for the DM and DN groups; the printed materials consisted mostly of questions. If a student asked the instructor for help, he was referred back to the questions in the printed materials. Students in the DM treatment were expected to make discoveries using the blocks, while the DN group was often asked to reason by analogy, using their knowledge of base ten numeration. On the other hand, maximum guidance was provided to the EM and EN groups. These expository materials presented the concepts in complete detail. An example of each type of problem was worked out and explained in these materials for the student, and if a student asked a question, the instructor answered it directly and completely.

Students took from 60 to 140 minutes to complete the treatment. While there were substantial time differences between individuals, the mean time spent by each treatment group was about the same, approximately 90 minutes. When a student finished, the instructor checked over his work to see that it was reasonably complete before administering the posttest.

The posttest consisted of three subtests. In the first subtest (QN), all problems were presented symbolically and no blocks were available. In the second subtest (QM), students were encouraged to use multibase blocks, and certain problems were presented as pictures of blocks. QN and QM were parallel tests and included problems involving changing bases and addition and subtraction of whole numbers in bases other than ten. The third test was a transfer test which covered problems dealing with multiplication, division, and rational numbers in other bases, chip trading, and applications. Four weeks later QN and QM were readministered as a retention test.

Results are currently being analyzed using a repeated measures ANOVA. Preliminary analysis indicates that there are probably significant level-of-abstraction main effects for subtests QN and QM and possibly some significant interaction. It appears that there are no significant differences due to the level of guidance.

Thursday
4:00-5:00 p.m. RESEARCH REPORTING SECTION: NO. 150 3C

Presider: E. G. BEGLE, Stanford University
Stanford, California

Reports: THE EFFECT OF RETEACHING MATHEMATICS ON DIFFERENT COGNITIVE LEVELS

JOSEPH W. GREEN, University of Western Australia
South Perth, West Australia

A COMPARISON OF TWO REVIEW STRATEGIES FOR REVIEWING MATHEMATICS

SALLY H. THOMAS, Indiana University
Bloomington, Indiana

SMALL GROUP REVIEW OF MATHEMATICS: A FUNCTION OF THE REVIEW ORGANIZATION, STRUCTURE, AND TASK FORMAT

BARBARA J. PENCE, Stanford University
Stanford, California

THE EFFECTS ON SEVENTH GRADE STUDENTS OF MODIFYING MATHEMATICS INSTRUCTION FOR RETEACHING

ROBERT I. WISE, National Institute of Education
Washington, D.C.

Reactor: NICHOLAS A. BRANCA, The Pennsylvania State University
University Park, Pennsylvania

THE EFFECT OF RETEACHING MATHEMATICS ON DIFFERENT COGNITIVE LEVELS

Joseph W. Green
University of Western Australia

Purpose

Advocates of mastery learning emphasize that many alternative methods of review must be made available to students. These methods have become known as mastery strategies. However, there has been little research done to see if all are necessary, or even if some are more effective than others. One purpose of this study was to compare the effects of two mastery strategies with two non-mastery strategies.

Critics of the mastery learning theory claim that it is simply a training methodology, satisfactory for skills at the lower cognitive levels, but no further. A second purpose of the study was to see if strategies that result in higher achievement at these lower cognitive levels do or do not improve performance on items designed to test analytic thinking. Finally, these results were compared with those of students who were given practice in analytic type items during the learning process.

Procedure

Since there are so many variables in the classroom situation, programmed materials were prepared to control the teaching variable (all students received the same quality of instruction), and to make it possible to isolate the effect of review strategies. The programmed text which developed the topic Factors and Primes, was written in three versions: an initial teaching unit - Form A; a parallel review version - Form B; and an interweaving of these two - Form AB. In all versions, the cognitive level of the explanations and exercises did not go beyond simple application.

Form A was used as the initial teaching unit for all Ss. The review strategies investigated were:

1. Studying written explanations of items missed on the chapter diagnostic test.
2. Strategy (1) plus specific review in Form B of material pertaining to the items missed on the diagnostic test.
3. Strategy (2) plus two to four analysis level items on each diagnostic test.

Ss in each of these three experimental treatments who missed more than 20% of the items on the initial diagnostic test for a section also took a second parallel diagnostic test and received solutions for items missed. This study also included two control treatments: *Ss* who

1. Worked through Form A only, took no diagnostic tests and did no review,

2. Worked through Form AB, took the complete diagnostic test at the end of each section, but did no review.

No general review strategy was included since a previous study showed that any difference between general review and no review favored the latter.

The pretest battery consisted of four cognitive scales and one attitude test. On the completion of the third section, an achievement test was administered. This test was subdivided into two scales: those items that tested material from the text and those items at a cognitive level beyond the text.

Subjects came from 6 sixth grade classes, 2 each from 3 schools in middle class communities. A table of random permutations was used to allocate the students in each class to one of the five treatments.

Results

ANOVA was used for statistical analyses. Results on each scale were analyzed separately.

Although the review treatments scored higher than the controls, there were no significant differences between any of the treatments on either scale of the achievement test. That is, specific review for items missed on the diagnostic test did not significantly affect either achievement score. Also, since there was no significant difference in achievement at lower cognitive levels, the question of whether differences on the lower cognitive scale would carry over to the higher cognitive scale could not be tested. Finally, practice on analysis questions failed to produce significant differences in achievement.

Recommendations

Results from this study indicated that these review strategies did not improve achievement. An alternative interpretation is that following a programmed text by further programmed materials brings little change in learning. Is the same true if the format of the review is changed? Or again, in this experiment, giving the solution resulted in no better performance, yet a very standard teaching procedure in mathematics is to give a test and always follow this by correcting errors. Would this show no change if investigated rigorously?

Comparison of treatment 3 with the two control groups produced the only consistently positive differences in favor of a review strategy ($p < .06$). That such a trend is ovservable in an experiment in which practice in analytic thinking was very secondary and continued over such a short time, may be only a matter of chance or it may be of importance. It does, however, suggest an additional direction for further investigation.

A COMPARISON OF TWO REVIEW STRATEGIES FOR REVIEWING MATHEMATICS

Sally Halstead Thomas
Indiana University

Purpose

Mastery learning has recently had several advocates as one way of individualizing instruction and insuring that each student understands the materials he studies. Under this strategy, the student is expected to achieve at a pre-designated level of mastery on each unit he studies. Time to achieve to this level is varied. In contrast, in the traditional classroom, all students study a given unit for the same length of time and achieve at various levels on that unit. Student performance on a unit depends on the "quality of instruction." This raises the question of finding effective curriculum treatments for the initial presentation of new material. Even with a good instructional program, many students do not achieve satisfactorily after studying a given unit once. Thus, a mastery strategy also depends on finding effective review, or reteaching procedures.

The primary research questions of this study were:

1. Is initial instruction using a semantic and didactic approach more or less effective than instruction using a developmental and figural approach?

2. Is review using the same instructional mode as the original approach more or less effective than review using a different mode?

3. Is there an aptitude treatment interaction between the NLSMA test Five Dots, which was designed to measure ability to read and follow instructions to perform a simple but unfamiliar mathematical operation, and the treatment variables?

Two further questions were investigated:

4. Did students who had not mastered a unit initially profit from reviewing that unit?

5. Did student opinion of the material studied and the methods used depend on the treatment received?

Procedure

Two parallel and equivalent versions of the three-unit programmed text Factors and Primes for intermediate grade students were used to make the comparisons. One version was primarily semantic and didactic; the other was more figural and developmental. Each unit contained an initial teaching segment and a review segment. Each review segment was written so that it could follow either initial segment, thus allowing four different text combinations to be formed: two versions reviewed using the same mode as was used in the original instruction and two reviewed using the other mode of instruction.

Each unit was accompanied by two parallel diagnositc tests. The subject studied the initial segment of a unit, took the first diagnostic test on the unit, reviewed the unit by studying more of the initial explanation or a different explanation, then took the second diagnostic test on the unit. At the completion of the study, he took an achievement test on the material covered. He also completed a three-item opinion survey about the study. At the beginning of the study, each student took four pre-tests measuring intelligence, ability to read and follow unfamiliar instructions, previous knowledge of the material to be covered, and ability to use the basic division algorithm.

The subjects were 101 students in four classes in three different schools in a predominately white, upper-middle-class school district. There were three fifth grade classes and one fourth grade class. Students were assigned randomly within each classroom to one of the four treatment groups.

Results

Analysis of variance was used to examine research problems 1,2,3, and 5. No significant differences due to treatment were found on achievement test scores on the following variables: initial form of instruction, method of review, their interaction, or the aptitude treatment interaction. The classroom variable was significant. Thus, the treatments did not differ on variables that affect learning but classrooms did. Treatment groups differed significantly on the opinion survey only on whether they preferred to review using the same or a different explanation; this difference was not due to either treatment variable.

For research problem 4, non-mastery of a unit was defined as scoring seven points or less out of ten points on the first diagnostic test on the unit. Mean scores of these students on the two diagnostic tests on the unit were compared using t-tests. Students who had not mastered a unit initially scored significantly higher on the second of two parallel diagnostic tests on the unit; however, the difference was less than one point and was not sufficient to justify the time needed for the review.

Recommendations

Since all instructional materials and feedback were in written form, the importance of reading may have masked any differences between the treatments used. The significance of the classroom variable was surprising and interesting. Although classroom is frequently significant in curriculum studies, this has usually occurred when classes were taught by different teachers and classrooms formed the unit of instruction. In this study, programmed instruction was used to avoid the teacher variable and to allow each treatment to be assigned randomly to students within each classroom. Adjustments made using pre-test scores showed that the classroom difference was not due to differences on general or mathematical aptitude or on previous knowledge of the material studied. The opinion survey indicated that classes did not differ significantly on enjoyment of the material, preferred review method, or preference for using programmed instruction. This leaves open the question of how classroom did affect achievement.

SMALL GROUP REVIEW OF MATHEMATICS: A FUNCTION OF THE REVIEW ORGANIZATION, STRUCTURE, AND TASK FORMAT

Barbara J. Pence
Stanford University

Purpose

This study was designed to compare the effects of small group and individual review on subsequent individual performance. Review is a common phase of mathematics instruction. The objective of review is the identification and correction of student learning difficulties. Although the effect of different review procedures is unclear, Bloom, in his writings on mastery learning, suggests that the best procedure seems to be small group study sessions of two or three students. Review in small groups presents the opportunity to vary the mode of instruction and increase the involvement of each individual in the correction and learning process.

Research only partially supports the advantages of the small group organization for review. Findings justify the conclusion that the performance of a small group exceeds the performance of a matched number of individuals. However, the effect of small group review on subsequent individual achievement is inconclusive. Two variables, (a) structure of the review - the guidelines of group procedures which are externally stipulated, and (b) format of the responses elicited on the post review test, may contribute to the inconsistent results.

Bloom's small group recommendations coupled with the small group research findings, produced three experimental questions:

1. What is the effect of a small group versus individual review?
2. What is the effect of structured versus non-structured small group review?
3. What is the effect of multiple choice versus a cross number response format for the small groups?

In each case, the criterion was individual performance at the end of a learning unit.

Procedure

Each question defined an experiment. All three experiments were administered concurrently on subsets of the same population. Ss came from four sixth grade classes, two classes from each of two schools. They were randomly assigned to treatments within classes. Every S received the same initial instruction for each section of a programmed unit on the mathematical topic of factors of whole numbers. For each section, review, specific to concepts missed on the section diagnostic test, was assigned according to one of four treatments. In treatment 1, the subjects reviewed in small groups by cooperatively working through specific pages in a parallel version of the text. They then completed a multiple choice post review diagnostic test. In treatment 2, the subjects received the same review structure as in treatment 1 and then completed a cross number format of the post review diagnostic test. Treatment 3 consisted of review in small groups without the additional instructional material, but questioning of an adult expert was encouraged. The subjects then completed the multiple choice post review diagnostic test. In treatment 4, the subjects received the same materials as in treatment 1, but all review was processed individually.

Four cognitive scales and seven attitude scales formed the pretest battery. Performance criteria were collected for each individual and included the same seven attitude scales and a summative achievement scale. The time required to complete the unit was also recorded.

Results

ANCOVA was used for statistical analysis of the achievement scale data. ANOVA was used for the time data because correlations with the pretests were not significant.

Since treatment 4 was administered in a previous study to similar Ss, data from the previous individual review sample was included in the analysis of the effect of small group versus individually organized review.

The anticipated achievement trends favoring the small group review were replicated in both the across and within study comparisons but the results were not significant. The within study comparison produced the only significant result; small group review required significantly more time, $p < .05$.

A second experimental comparison, structure of the small group review, treatment 1 versus treatment 3, produced no significant achievement or attitude differences. Time means were significantly different: the open structure, treatment 3, required significantly less time. Adjusted achievement means for treatment 3 were also higher in each school.

Experimental comparison three, format of the group task, treatment 1 versus treatment 2, failed to produce any significant differences.

Finally, although the between treatment attitude differences were not significant, the common pre-post decrease in attitude means across treatments appeared and were significant.

Recommendations

Implications for further research lie in the replicated trends. The comparison of small group and individual review was inconclusive; but, trends favoring small group review provided a good case for continuing this line of research. Trends favoring the open structure also motivated conjectures for additional investigations. It may be possible that an interaction exists between the level of the examples encountered in the initial experience with the material and the degree of specificity required in the small group review. It could be hypothesized that with these materials, which require only routine computation and application of text concepts, review structure in addition to the diagnostic tests retards and inhibits the group's potential for correcting deficiencies of its members. On the other hand, for materials which include non-routine problem solving as well as the lower cognitive levels, the specific review may be a necessary supplement to the knowledge of the individual group members.

THE EFFECTS ON SEVENTH GRADE STUDENTS OF MODIFYING MATHEMATICS INSTRUCTION FOR RETEACHING

Robert I. Wise
National Institute of Education

Purpose

The purpose of this study was to explore the effects of modifying the presentation of an unsuccessful lesson for reteaching. In this study, "reteaching" is a term given to a common classroom practice used by teachers to correct unsuccessful lessons. Reteaching is used when an earlier presentation of a lesson has not brought about successful student learning. The assumption to be made in reteaching is that a student is not lacking the prerequisite knowledge for successful learning, only that he has not learned from the earlier presentation. Reteaching is an adaptive procedure which provides additional instruction when students need it.

Previous research offered little help in deciding what modifications to include in this study. Four types of modifications that a teacher might customarily think of making and that could be argued as having possible effectiveness were selected. These four modifications were: (1) increasing the redundancy of explanations, (2) using maximally different examples and exercises, (3) changing the sequence in which the content is presented, and (4) changing the mode of representing content.

Procedure

To control for teacher and class effects, all instruction in this study was presented via programmed instructional booklets. The subject matter was an introductory unit in probability theory. Five booklets were used, one (Booklet A) to make the initial presentation of the unit, the other four (Booklets B,C,D, and E) used in reteaching to represent various alternative ways of modifying Booklet A.

The four modifications were grouped into two sets of two and arranged in a 2 x 2 experimental design. The first set of modifications included (1) increasing the amount of redundancy and (2) using maximally different examples and exercises. The second set included (3) changing the sequence and (4) changing the mode of representation. The first alternative, Booklet B, contained no changes from Booklet A except to provide new practice exercises. Booklet C contained the first set of modifications, while Booklet D contained the second set. Booklet E contained both sets of modifications.

Subjects were 78 students in four seventh grade mathematics classes. They were randomly assigned to treatments within classrooms. Each student studied Booklet A and one of the alternative booklets. Data on the students' mathematics aptitude and their general attitudes toward mathematics were obtained at the beginning of the study.

Four measures were used to determine the comparative effects of the four reteaching treatments: student achievement, student attitude toward how easy the subject matter was (Easy), student attitude toward how enjoyable the subject matter was (Enjoy), and student disposition toward further study of the subject matter (More). Data on achievement, Easy, and Enjoy were gathered both before and after reteaching. Pre and post achievement data were obtained through parallel forms of an achievement test. Pre and post data on Easy and Enjoy were obtained by administering a semantic differential before and after reteaching. Three adjective pairs represented an easy-hard dimension, and two adjective pairs represented an enjoyable-boring dimension. The variable More was measured only once at the end of the study. The various scale reliabilities ranged from .76 to .90.

Results

Seven analyses of the data were carried out: one MANOVA of the four post-reteaching variables (achievement, Easy, Enjoy, More), four univariate ANOVA's of the same four variables taking pre-reteaching scores into account in the case of achievement, Easy, and Enjoy, and two ANCOVA's which were done as alternative analyses of Easy and Enjoy.

The MANOVA used a 2^5 design and a sample of 64 students randomly selected from the total sample of 78. The design contained the 2 x 2 design for treatment conditions and three blocking variables: class, entering mathematics aptitude, and entering mathematics attitude. Each blocking variable contained two levels, "high" and "low."

The four ANOVA's were carried out in order to locate specific treatment effects. In the case of the three measures with pre- and post-reteaching data (achievement, Easy, Enjoy), pre and post scores were handled as two levels of a within-subjects factor. Analysis of the variable More had no within-subjects factor.

The two ANCOVA's of Easy and Enjoy used data from all 78 students and used a one-way design to represent the four treatment conditions. In each ANCOVA, the covariate represented the pre-reteaching scores on the variable analyzed.

The multivariate analysis of the four post-reteaching measures produced significant F values ($p < .01$) for entering mathematics aptitude and for the interaction of the treatment main effects.

The univariate analyses showed no differential treatment effects for achievement or for disposition toward further study of the subject matter. A significant difference due to the interaction of treatment main effects was found for each of the two attitude measures, how easy ($p < .05$) and how enjoyable ($p < .01$) the subject matter was, but these differences were not attributable to the effects of reteaching. The results of analyzing these same two variables (Easy and Enjoy) using ANCOVA was completely consistent with this interpretation.

Recommendations

It was concluded that the modifications made for reteaching had no differential effects on student achievement, attitudes toward the subject matter, or disposition toward further study of the subject matter. It was noted that, regardless of treatment, no statistically significant change occurred in achievement or attitudes from pre to post measures as a result of reteaching. That is, providing additional instruction seemed to have little or no effect in this study. There was some gain in achievement, but it was not statistically significant. Although the expected positive relationship between amount of instruction and amount of learning is generally observable over the long run, its observability in the short run is questioned by this study.

Friday
10:00-11:00 a.m. RESEARCH REPORTING SECTION: NO. 185 3C

Presider: LESLIE P. STEFFE, University of Georgia
Athens, Georgia

Reports: DIAGNOSIS AND REMEDIATION IN MATHEMATICS FOR ENTERING FIRST GRADE STUDENTS

MORRIS I. BEERS, State University of New York
Brockport, New York

AN INVESTIGATION OF CHILDREN'S KNOWLEDGE OF THE MEANINGS OF SELECTED SYMBOLS ASSOCIATED WITH MULTIPLICATION OF WHOLE NUMBERS

DENNIS FORDHAM, Shorter College
Rome, Georgia

THE DESIGN AND EVALUATION OF A UNIT ABOUT MEASUREMENT AS A VEHICLE FOR CHANGING ATTITUDE TOWARD MATHEMATICS AND SELF-CONCEPT FOR LOW ACHIEVERS IN INTERMEDIATE GRADES

HELENE SILVERMAN, Herbert H. Lehman College
Bronx, New York

A STUDY OF CHILDREN'S ABILITY TO USE INFERENCE PATTERNS WITH MATCHING AND LENGTH RELATIONS

CHARLES E. LAMB, University of Georgia
Athens, Georgia

LESLIE P. STEFFE, University of Georgia
Athens, Georgia

DIAGNOSIS AND REMEDIATION IN MATHEMATICS FOR ENTERING FIRST GRADE STUDENTS

Morris I. Beers
State University of New York

Purpose

Bloom (1964) has indicated that nearly 50 percent of all growth in human intelligence takes place before a person reaches his fourth birthday and an additional 30 percent before he has completed the second grade. If these estimates are accurate, instructional time during early childhood years should be put to the best possible use.

There is much to indicate that if a certain level of readiness is reached prior to presentation of new material, learning will be greatly facilitated. According to Copeland (1970), "it is very likely that teachers are 'teaching' mathematical ideas (especially in kindergarten through third grade) before the child can understand them (p.57)." Piaget (1964) feels that teachers should be aware of the stage-development in children because they often teach (particularly in first and second grade) in such a fashion that many students survive only by rote memorization and that understanding will not occur until much later, if ever.

Readiness is a key to successful early experiences in mathematics. It seems reasonable to assume that, in order to diagnose a particular level of readiness, tests should be employed which will enable teachers to detect potential areas of difficulty. Once these areas were located, supplementary materials could be used to bring the student to the proper level of readiness before introducing the normal first grade material. This process of remediation at the readiness level should be more productive, for both student and teacher, than remediation of poorly learned material after the fact.

This paper describes a two-fold exploratory study. The first purpose was to assess the ability of the previously prepared Peabody Mathematics Readiness Test (1973) to detect the need for remediation in entering first grade students. The second, and perhaps more important purpose of this investigation, was to ascertain the effectiveness of simple remedial materials on student readiness. The assessment of the remedial material was based on the comparison of pretest and posttest scores on parallel forms of the Peabody Mathematics Readiness Test as well as on parallel forms of the Metropolitan Readiness Test (1959).

Procedure

The subjects for this investigation were seventy-two students (47 boys and 25 girls) drawn from four kindergartens in Nashville, Tennessee. The children attending these four schools were from middle- to upper-middle income families, and could be considered to be of average general intelligence.

There were two pretests and two posttests administered for the study. The posttests were parallel forms of the pretests. The Peabody Mathematics Readiness Test consists of two parts, a five item drawing test and a forty-one item multiple dimension discrimination test. Each of the forty-one items in the discrimination portion of the test was classified into one of five subscales: (1) Number, (2) Shape, (3) Containment, (4) Size, and (5) Configuration (including symmetry). Each subscale was previously assigned a weight through the application of multiple linear regression techniques.

The training consisted of units dealing with the following mathematical concepts: (1) seriation, (2) ordination, (3) cardination, (4) one-to-one correspondence, (5) classification, (6) reversibility, and (7) conservation. These concepts were selected as basic to good understanding of first grade mathematics and comparatively immune to the interpretation that training on these concepts would constitute training for the type of items that appear on the Peabody Mathematics Readiness Test. In accordance with Bruner (1966), these tasks started with a concrete or manipulative phase where the subjects were allowed to handle the materials and proceeded, whenever possible, to a semi-concrete (semi-abstract) phase where the subjects were presented with pictures and diagrams rather than manipulative materials.

All subjects were given the Peabody Mathematics Readiness Test along with the mathematics section of the Metropolitan Readiness Test. Both tests were administered during the week of March 12, 1973. On the basis of the Peabody Mathematics Readiness Test, those students indicated as needing remediation were identified using the previously developed regression equation. After all subjects assumed to need remediation were identified, they were randomly assigned to either the control or the experimental group.

Once the groups were constituted, the training sessions began. The group designated for remediation (experimental) received eight one-half hour training sessions given by the investigator. The sessions were given two days per week, during which time control groups from each school spent working with their regular teacher in language arts, art, physical education, etc.

At the end of the eight sessions parallel forms of the Peabody Mathematics Readiness Test and the Metropolitan Readiness Test were administered and the results analyzed.

Results

1. Metropolitan Readiness Test: The correlation between pretest and posttest on parallel form of the Metropolitan Readiness Test was 0.72.

According to Feldt (1958, p. 347), when the correlation between the covariate and the criterion variable exceeds 0.6, analysis of covariance should be used since it will appreciably increase precision over a 3-factor analysis of variance design. A 2-factor analysis of covariance permitted the investigation of main effects as well as the interaction between treatment and schools.

When the criterion scores were adjusted by covarying out the pretest scores, there was no significant interaction between treatment and schools and no significant school effect, but there was a significant treatment effect.

Those students in the treatment group performed significantly better than those in the control group.

2. Peabody Mathematics Readiness Test: The correlation between the pretest and posttest scores on the parallel forms of the Peabody Mathematics Readiness Test was 0.72; therefore, a two-factor analysis of covariance was used. When the criterion scores were adjusted by covarying out the pretest scores, there was a significant interaction between treatment and schools.

Secondary analyses were run on the Peabody Mathematics Readiness Test and results indicated that the interaction was attributable to differential treatment effects across schools.

Recommendations

Considering the findings of the analysis of covariance for the Metropolitan Readiness Test, within the limitations imposed by the sample population, it was concluded that a program of special training can improve student achievement. Exposure to simple Piagetian concepts during the kindergarten year was apparently worthwhile because of the significant increase on the Metropolitan Readiness Test scores. Since the lower one-half of each class showed an increase in achievement, another conclusion of this study was that the inclusion of training on such concepts was indicated, at least for some students.

The format of the instruction was also found to be important. The subjects seemed to enjoy being presented with problems, rather than being shown what to do. Children who were on the borderline between the pre-operational and concrete operations stages seemed to benefit from the combination of manipulative and semi-abstract (paper and pencil) tasks. This was due mostly to the fact that the transtition from manipulative to semi-abstract was made within the confines of the same basic concept, so the transition was a natural one.

In short, it is concluded that a program of this type for kindergarten or the beginning of first grade would be beneficial. The test would be an important part of the program in that it would allow grouping according to readiness levels.

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AN INVESTIGATION OF CHILDREN'S KNOWLEDGE OF THE MEANINGS OF SELECTED SYMBOLS ASSOCIATED WITH MULTIPLICATION OF WHOLE NUMBERS

Dennis Fordham
Shorter College

Purpose

The purpose of this study was to explore the nature of meaning and meaningful learning in mathematics, in the context of multiplication of whole numbers. To facilitate the investigation, an operational definition of the term "meaning" was presented. The position taken in the study was that any meaningful situation in mathematics involves a symbol, a referent, and an individual to interpret the symbol as referring to the referent. For any mathematical symbol, there are certain appropriate actions that serve as referents for the symbol. A meaning of a symbol was defined as a pairing, in the mind of an individual, of the symbol with some appropriate action. A meaning was said to be semantic if the action paired with the symbol involved concrete or semi-concrete materials. A meaning was said to be syntactic if the action paired with the symbol involved only other symbols.

Specifically, the exploration of the nature of meaning was carried out through the investigation of eight research questions:

1. If an individual knows a meaning of one symbol of the form a x b, then does he necessarily know a meaning of any other such symbol?

2. If a student is able to suggest multiplication as a means of determining "how many" in some appropriate situation, then does he necessarily know a meaning of symbols of the form a x b?

3. If a student knows a meaning of symbols of the form a x b, then will he necessarily be able to suggest multiplication as a means of determining "how many" in some appropriate situation?

4. If a student knows a meaning of symbols of the form a x b, and if he has good habituation of the basic multiplication facts, then will he necessarily be able to suggest multiplication as a means of determining "how many" in some appropriate situation?

5. Are students who know both semantic and syntactic meanings of symbols of the form a x b, compared with students who know only one kind of meaning, more likely to (a) use multiplication to determine "how many" in appropriate situations, and (b) solve story problems calling for the use of multiplication?

6. Are students who know only semantic meanings of symbols of the form a x b, compared with students who know only syntactic meanings, more likely to (a) use multiplication to determine "how many" in appropriate situations, and (b) solve story problems calling for the use of multiplication?

7. Do students who know a meaning of symbols of the form a x b perform significantly better than students who do not know a meaning of such symbols when asked to solve story problems calling for the use of multiplication?

8. Do students who are able to suggest multiplication as a means of determining "how many" in some appropriate situation perform significantly better than students who are not able to suggest multiplication when asked to solve story problems calling for the use of multiplication?

Procedure

The 88 subjects in the study were enrolled in the third, fourth, and fifth grades of the David C. Barrow Elementary School in Athens, Georgia. The data for the study were collected in interviews conducted by the investigator with each of the subjects. Each interview consisted of three different parts.

In the first part of the interview, the subject was asked to solve story problems calling for the use of multiplication. During the second part, the subject was presented with 15 symbols of the form a x b, where a and b are whole numbers. The subject was questioned on each symbol to determine if he knew a meaning of the symbol, using the definition of "meaning" given above. If he knew a meaning of the symbol, then the meaning was classified as either semantic or syntactic. In the third part of the interview, the subject was questioned to determine if he could interpret various counting situations (e.g., a rectangular array made from small ceramic tiles) as being ones for which multiplication is an appropriate means of determining "how many."

The statistical analysis of the data for each of the research questions Q-1, Q-2, Q-3, and Q-4 consisted of the determination of a 95% confidence interval for the probability that the answer to the question is "yes." Each of these questions was investigated on three levels, based on three sets of symbols of the form a x b. One of these sets of symbols included only one-digit whole numbers, with no 0's or 1's. A second set included one-digit whole numbers, with 0's and 1's. A third set included both one-digit and two-digit whole numbers, with no 0's or 1's.

Results

Space does not permit the reporting of the results of the analyses of all of the first four questions. The results of the analysis of Q-1 is given below, to serve as an example of how the results were reported.

For question 1, the results of the analysis were (with p = probability that for a given student either all symbols are meaningful or all are not meaningful):

(1) for symbols of the form a x b that involve only one-digit whole numbers, with no 0's or 1's, it can be stated with a confidence of .95 that $.89 \leq p \leq .99$;

(2) for symbols of the form a x b that involve one-digit whole numbers, including 0's and 1's, it can be stated with a confidence of .95 that $.58 \leq p \leq .78$;

(3) for symbols of the form a x b that involve one-digit and two-digit whole numbers, with no 0's or 1's, it can be stated with a confidence of .95 that $.55 \leq p \leq .75$.

The analysis of data for each of the research questions Q-5, Q-6, Q-7, and Q-8 was accomplished through the use of a chi-square test for difference in probabilities. Space does not permit the reporting of the results of the analyses of all of these last four questions. The results of the analysis of Q-8 is given below, to serve as an example of how the results were reported.

For question Q-8, a chi-square test for difference in probabilities was used to test the null hypothesis that ability to solve story problems calling for the use of multiplication is the same for students who are and for students who are not able to suggest multiplication as a means of determining "how many" in some appropriate situation. This null hypothesis was rejected, in favor of the students who were able to suggest multiplication, with almost negligible probability ($< .001$) of type I error.

Recommendations

Conclusions based on the analyses of some of the other questions included the following:

1. Students who know both semantic and syntactic meanings of symbols of the form a x b, compared with students who know only one of these kinds of meaning, are more likely to use multiplication to determine "how many" in an appropriate situation.

2. If a student is able to suggest multiplication as a means of determining "how many" in some appropriate situation, then he knows a meaning of symbols of the form a x b that involve only one-digit whole numbers, with no 0's or 1's. This also appears to be true for symbols involving 0's and 1's and two-digit whole numbers, but the evidence is less conclusive.

DESIGN AND EVALUATION OF A UNIT ABOUT MEASUREMENT AS A VEHICLE FOR CHANGING ATTITUDE TOWARD MATHEMATICS AND SELF-CONCEPT FOR LOW ACHIEVERS IN THE INTERMEDIATE GRADES

Helene Silverman
Herbert H. Lehman College

Purpose

The basic drive of the individual is the maintenance and enhancement of self and a constant and organized view of the world (Jersild, 1956; DeLevita, 1965; Maier, 1965; Brandon, 1969; LaBenne and Green, 1969; Piaget, 1970; Purkey, 1970).

All behavior depends on the individual's frame of reference: things are either valuable or worthless, important or unimportant, attractive or unattractive, interesting or dull, in terms of their relevance to the individual (Rogers, 1961; Purkey, 1970). "The way an individual views himself and his world are products of how others see him and primary forces in his academic achievement (Purkey, 1970, p. 2)." A student who has done well in the past expects to do well in the future. He enters each new situation with eagerness and confidence. Setbacks are only temporary. On the other hand, a student with a history of failure expects to do poorly. Any setback confirms his negative self-image (Bernstein, 1964).

In addition, a student may do poorly in school because what he is doing may seem irrelevant to him and his world (Fantini and Weinstein, 1966; Purkey, 1970). Dutton (1956, 1968) found that students identified story problems and lack of utility as reasons for disliking mathematics.

Bernstein (1964) cited feelings of success from extrinsic rewards or intrinsic satisfactions, the appeal of linear structure and logic in mathematics, applications of mathematics in the physical and social world, and relationship to a teacher as causes for motivation in mathematics. Feelings of failure, rote learning, story problems, poor relationship with a teacher, cultural stereotyping of mathematics as difficult, and emotional disorders are causes for lack of motivation.

According to Kidd, Meyers, and Cilley (1971), the low achiever displays many of the characteristics classified by Bernstein as reasons for disliking mathematics. He has seldom enjoyed doing mathematics, he prefers to work with objects rather than abstractions, he requires recognition, and he needs help organizing ideas (Bernstein, 1964; Kidd, Meyers, and Cilley, 1971).

The present investigator reviewed the New York City Syllabus for grades four, five, and six. After carefully analyzing the recommendations of the Cambridge Conference on School Mathematics (1963), the Intermediate

Schools Task Force Report (1966), and the Cambridge Conference on the Correlation of Science and Mathematics in the Schools (1969), the present investigator developed a unit, About Measurement.

Measurement was chosen because of the possibility of including self-measurement and utilizing the student's concerns about himself in relationship to other people. Measurement as a topic for low-achievers has been suggested by Holt (1970), Marsh (1970), Gattegno (1970), Kidd, Meyers and Cilley (1971) and many advocates of the British primary style of teaching as described in the Plowden Report (1970).

The purpose of this study was to design and evaluate a unit about measurement as a means of effecting positive changes in attitude toward mathematics and self-concept for low-achievers in the intermediate elementary school grades. The study focused on the following variables: attitude toward mathematics, self-concept, sex, grade level, and mastery of the unit.

Specifically, the investigation sought to answer the following questions:

1. To what extent can a unit about measurement effect a positive change in attitude toward mathematics?
2. To what extent can a unit about measurement effect a positive change in self-concept?
3. To what extent is there a relationship between change in attitude toward mathematics and mastery of a unit about measurement?
4. To what extent is there a relationship between change in self-concept and mastery of a unit about measurement?
5. To what extent is there a relationship between change in attitude toward mathematics and change in self-concept?
6. To what extent are there differences among the variables on the basis of grade level?
7. To what extent are there differences among the variables on the basis of sex?

Procedure

The subjects in this investigation were 684 low-achieving students in the fourth, fifth, and sixth grades in five Title I New York City public schools. Five schools were chosen, three from one school district and two from another.

The materials used in this study were: (1) the unit, About Measurement, (2) a Teacher's Manual, (3) Activity Sheets, (4) Help Sheets, (5) a Unit Test, (6) the Teacher's Rating Scale, (7) the Attitude Questionnaire, and (8) the Parker About Me scale.

The investigation involved a pilot study, a field experiment, and a four-part statistical study. A pilot study was conducted with two fifth-grade classes in February and March, 1972. Thirty classes participated in the field testing during May and June, 1972. The statistical procedures utilized in this investigation were: (1) comparison of the pretests, (2) comparison of pretests and posttests, (3) analysis of covariance of major effects and interactions, and (4) Chi-square analysis of Unit Test scores.

To meet the needs of the students, lessons focused upon active involvement. Students were taught to find large and small measurements, to record information, to compare information using graphs, to estimate, and to interpret data from graphs and tables. Students were involved in planning and goal setting. Structure within the physical world and the historical development of some of the common units of measure were deliberately stressed so that the student could relate to the greater world. Whenever possible, lessons were related to self-measurement. There were deliberate attempts to insure successful experiences. Attitudes and competencies were stressed rather than facts. A variety of approaches and answers was encouraged.

Results

Analysis of the data led to the following conclusions: (1) a unit about measurement can effect a positive change in attitude toward mathematics; (2) a unit about measurement can effect a positive change in self-concept; (3) attitude toward mathematics is directly related to mastery of a unit about measurement; (4) self-concept is directly related to mastery of a unit about measurement; (5) there seems to be a direct relationship between attitude toward mathematics and self-concept; (6) there were significant differences among the interaction effects on the basis of grade level; and (7) there were significant differences among the interaction effects on the basis of sex.

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A STUDY OF CHILDREN'S ABILITY TO USE INFERENCE PATTERNS WITH MATCHING AND LENGTH RELATIONS

Charles E. Lamb
University of Georgia

Leslie P. Steffe
University of Georgia

Purpose

Logical reasoning in young children has received attention during recent years by mathematics educators. In particular, transitive reasoning has been the topic for several investigations (Johnson, 1971, Owens and Steffe, 1972, and Owens, 1972). Experimenters, however, have not focused on other aspects of logical reasoning to the same extent as they have transitivity and syllogistic reasoning involving the conditional and biconditional. This study was concerned with logical reasoning involving transitivity, the substitution property, and premises from which no conclusion can be drawn (incompatible premises). The relational categories of interest were matching relations (as many as, more than, and fewer than) and length relations (as long as, longer than, and shorter than).

Relations are, of course, an important mathematical entity in themselves but their involvement with the topics of number and measurement makes them even more important as a subject of investigation. Only through being operational with the matching and length relations will a student have the foundation for future study in number and measurement. Essential to this operativity is a working knowledge of transitivity, substitution and incompatible premises.

Smedslund (1963) has argued that in order to assess concrete reasoning ability, one must make a clear distinction between stimulus percept, goal object, and inference pattern. Stimulus percept is that set of properties inherent in the physical situation as presented to the child. Goal object is that which the subject is told to obtain, for example, a length or matching comparison. Inference pattern is a set of premises and a conclusion.

In the present study, the stimulus percept was held constant within relational categories. Across each task, no attempt was made to deceive the child. A neutral stimulus was employed. Goal objects were of two types, the matching and length relations. Inference patterns under consideration were transitivity, substitution, and incompatibility. In the incompatible items, the child was confronted with situations where no logically necessary conclusion could be drawn.

Procedure

Each child received eighteen tasks on matching relations and eighteen on length relations. Two items were presented in each relational category for each of the nine paradigms:

(1) If $A = B$ and $B = C$, then $A = C$.
(2) If $A < B$ and $B < C$, then $A < C$. Transitivity
(3) If $A > B$ and $B > C$, then $A > C$.

(4) If $A = B$ and $B < C$, then $A < C$.
(5) If $A = B$ and $B > C$, then $A > C$.
(6) If $A < B$ and $B = C$, then $A < C$. Substitution
(7) If $A > B$ and $B = C$, then $A > C$.

(8) If $A < B$ and $B > C$, then _____.
(9) If $A > B$ and $B < C$, then _____. Incompatibility

Within each of the relational categories, a child was assigned a score of 0, 1, or 2 for each inference pattern. In order to score two points the child had to make correct inferences in more than half the possible cases and also give rational justifications for at least half the tasks. A score of one point was received when a child could make more than half of the possible correct inferences but was unable to make at least half of the justifications. Any performances below these levels was assigned a score of zero.

Results

Scores were recorded and used in a repeated measures design (Winer, 1971). Repeated measures were obtained on goal object and inference pattern. The summary ANOVA is presented below:

Source	Ss	df	ms	F	
Between Subjects	62.42	49			
G (grade level)	19.28	4	4.82	5.02*	$p < .01$
Subj. w. groups	43.14	45	.96		
Within Subjects	127.33	250			
R (Goal objects)	2.26	1	2.26	9.83*	$p < .01$
GR	1.88	4	.47	2.04	
R x subj. w. groups	10.52	45	.23		
I(Inference patterns)	49.93	2	24.97	62.43*	$p < .01$
GI	2.64	8	.33	.83	
I x subj. w. groups	33.76	90	.40		
RI	4.12	2	2.06	8.96*	$p < .01$
GRI	1.64	8	.21	.91	
RI x subj. w. groups	2.58	90	.23		

Because of the significant interaction between goal object and inference pattern, a Newman - Keuls test (Winer, 1971) was performed. A table showing significant differences ($p \leq .01$) is presented below.

R_1 Matching Relations
R_2 Length Relations
I_1 Transitivity
I_1 Substitution
I_3 Incompatibility

Table of Means

$\overline{R_1\ I_1} = 1.42$

$\overline{R_1\ I_2} = 1.34$

$\overline{R_1\ I_3} = .76$

$\overline{R_2\ I_1} = 1.36$

$\overline{R_2\ I_2} = 1.38$

$\overline{R_2\ I_3} = .26$

Significant Differences

$\overline{R_1\ I_1}$, $\overline{R_2\ I_2}$, $\overline{R_2\ I_1}$, $\overline{R_1\ I_2}$ are not significantly different
↓ ↓
$\overline{R_1\ I_3}$ significantly different
↓ ↓
$\overline{R_2\ I_3}$ significantly different

Grade level proved to be a significant factor, with older (in grades K-4) children performing at higher levels. The post-hoc analysis of the significant interaction between goal object and inference pattern showed differences existing when the child was involved with the incompatible category. In this case, there existed a difference between the goal objects. Matching relations with incompatibility was more difficult than most tasks but length relations with incompatibility proved to be the most difficult of all.

Recommendations

Conclusions may be summarized as follows:

1. Ability to use inference patterns across goal objects is age (grade level) related.

2. When one considers the incompatible category (apparently the most difficult inference pattern), there is a difference in terms of goal objects. This difference is in favor of matching over length relations. This is accounted for by the fact that in matching tasks discrete objects were used, whereas in length tasks continuous objects were utilized.

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Friday
1:00-2:00 p.m.   RESEARCH REPORTING SECTION: NO. 241   3C

Presider: J. FRED WEAVER, University of Wisconsin
Madison, Wisconsin

Reports: A COMPARISON OF TWO INSTRUCTIONAL LESSONS FOR TEACHING DIVISION INVOLVING ZERO

DOUGLAS A. GROUWS, University of Missouri
Columbia, Missouri

ROBERT E. REYS, University of Missouri
Columbia, Missouri

AN ANALYSIS OF THE FRACTION CONCEPT INTO A HIERARCHY OF SELECTED SUBCONCEPTS AND THE TESTING OF THE HIERARCHICAL DEPENDENCIES

CAROL F. NOVILLIS, Florida International University
Miami, Florida

THE RELATIONSHIP BETWEEN ADOLESCENT LOGICAL REASONING ABILITY AND NATURAL AND MODELED UTILIZATION OF THE LANGUAGE OF LOGIC

JOHN W. GREGORY, University of Florida
Gainesville, Florida

THE EFFECT OF TRAINING FOURTH GRADE, SEVENTH GRADE, AND HIGH SCHOOL PUPILS TO PERFORM COLUMN ADDITION DIRECTLY ON USING TENS

GRAYSON H. WHEATLEY, Purdue University
Lafayette, Indiana

DANIEL O. MCHUGH, Seeger High School
West Lebanon, Indiana

A COMPARISON OF TWO INSTRUCTIONAL LESSONS FOR TEACHING DIVISION INVOLVING ZERO

Douglas A. Grouws
University of Missouri-Columbia

Robert E. Reys
University of Missouri-Columbia

Purpose

There were two primary purposes for conducting the research: (1) to gather base-line data concerning pupils' performance in solving certain division problems, some involving zero and others not involving zero, and (2) to evaluate the effectiveness of two instructional strategies for teaching division involving zero.

Division is an important topic in the elementary school mathematics curriculum and division involving zero seems to cause out-of-the-ordinary difficulty for students. Most previous research in this area has focused on division algorithms where attention was not directed toward zero as a divisor or dividend. Thus, this research provides data which helps bridge a knowledge gap and at the same time provides useful instructional implications.

In answering questions associated with the previously mentioned purposes, the study is based on and contributes to three major theoretical lines of research. First, the study takes into account and contributes to theory associated with meaningful learning. Second, the research examines certain popular positions concerning learning in a symbolic mode. Third, information about the forming of generalizations and the related concept of overgeneralization is reported.

Procedure

The study was conducted over three consecutive school days. On the first day a pretest was administered to ascertain prior knowledge of division. On the second day, instruction on division was given and practice exercises were completed and discussed. On the third day each student took a posttest. A retention measure was collected six weeks after the posttest.

Over 700 students from a total of 30 classes from four school districts distributed over the fourth-, sixth-, and eighth-grade levels comprised the sample for the study. Two different instructional lessons based on the inverse relationship between multiplication and division were developed. The M-D lesson focused on solving a multiplication sentence and then solving the related division sentence. After a number of such pairs of sentences were solved the children were asked to look for patterns and this then led to establishing the inverse idea. Division examples involving zero were then considered by first examining an appropriate multiplication sentence (e.g., $8 \times \square = 0$) and then ascertaining the solution to the related division sentence (e.g., $0 \div 8 = \square$) through the use of the inverse relationship. The other lesson, the D-M lesson, was quite similar to the first lesson except that consideration was always given first to a division sentence and then the related multiplication sentence.

One-half of the classes at each grade level in each district were taught division involving zero using the M-D lesson and the remaining one-half used the D-M lesson. All teaching and testing was done by the investigators.

Three parallel forms of an 18-item test were developed and used. The tests were designed so that solving performance on items having zero as the divisor, zero as the dividend and zero not involved were equally represented. Test items were written in three notational forms ($\frac{a}{b}$, $b\overline{)a}$, and $a \div b$). On each test form the 18 items were crossed and balanced with regard to the zero factor and the notational factor.

Test-retest reliability for the three forms was high ranging from .84 to .95.

Analysis

The data were analyzed using analysis of covariance with pupil pretest scores as the covariate. The factorial design included four factors: Instructional Treatment (2 levels), Grade (3 levels), School District (4 levels), and Sex (2 levels). Descriptive statistics were used to examine performance on individual test items.

Results

The pretest mean score was 10.4, the posttest mean score was 11.9, and the retention mean score was 11.2.

The ANCOVA on posttest scores resulted in the rejection ($p < .0008$) of the null hypothesis of no effects due to instructional treatment. The adjusted mean for the D-M treatment group was 12.3 and the adjusted mean score for the M-D treatment group was 11.4.

There also was a significant difference ($p < .0001$) associated with the grade level variable. The adjusted means for each grade level were: grade 4 (11.0), grade 6 (12.2), and grade 8 (12.4). The null hypothesis concerning sex difference was not rejected ($p = .9704$); the adjusted mean for boys was 11.88 and the adjusted mean for girls was 11.89. In addition, the district by treatment and district by grade interactions were significant ($p < .0012$ and $p < .0009$, respectively).

ANCOVA was also carried out on retention test scores with pretest scores as the covariate. Main effects associated with grade ($p < .0002$) and district ($p < .0001$) were significant but the main effect associated with instructional treatment was not significant ($p < .53$).

Conclusions

The study demonstrated that some children at each of the grade levels studied can successfully learn to work with division involving zero. Greater success, however, seems to be achieved with sixth and eighth grade pupils. This suggests that children at the latter age levels can learn division ideas involving zero which are presented in a symbolic yet meaningful way. The extent to which this holds for other topics is worthy of further study. The fact that the learning taking place was, in many cases, meaningful rather than rote, was supported by interview data collected but not presented here. A caution is also in order since the interview data showed that a number of children did overgeneralize and assume that all quotients for division exercises involving zero were undefined, or alternatively, all such quotients were zero.

The study also showed that a teacher's choice of instructional lessons may affect student achievement. Based on the results of this study, a good instructional approach to teaching division involving zero might first focus on the division sentence to be solved and then focus on writing the related multiplication sentence to help in solving the division sentence.

The retention data suggest that periodic maintenance and review are necessary if learning gains associated with instruction on division involving zero are to be sustained. Of particular interest and worthy of further study, is the question of whether the differences between the D-M and M-D treatments at the time of the posttest would be maintained over time if periodic review and maintenance were included in an instructional plan.

AN ANALYSIS OF THE FRACTION CONCEPT INTO A HIERARCHY OF SELECTED SUBCONCEPTS AND THE TESTING OF THE HIERARCHICAL DEPENDENCIES

Carol F. Novillis
Florida International University

Purpose

Many elementary school students have difficulty in developing a fraction concept and in learning to compute with fractional numbers. The extensive work of Piaget suggests that concepts which in the past were considered to be simple are in reality extremely complex and develop slowly over a number of years (Lovell, 1966). The problem addressed in this report focuses on the complexity that the fraction concept possesses and the need for its analysis in order that all necessary defining characteristics be identified that can be taught to elementary school students. This focus is in agreement with Engelmann (1969) who lists concept analysis as the first step in developing teaching strategies for purposes of teaching concepts.

Based on Gagné's (1965) procedure for developing a hierarchy, the fraction concept was analyzed into a set of sixteen selected subconcepts which were arranged into a hypothesized hierarchy. Each subconcept was depicted by a model that had as its unit one of the following: a geometric region (part-whole model), a set (part-group model), or a unit segment of a number line (a specific form of part-whole model). The rationale for each dependency in the hypothesized hierarchy was based on, (1) mathematical development, (2) sequence in elementary mathematics textbooks, and (3) the limited research that has been done on fraction concepts.

The purpose of the investigation was to test the following hypothesis: If a student has attained criterion on the subtest associated with a superordinate subconcept, then he has also attained criterion on the subtest associated with each immediate subordinate subconcept.

Procedure

A Fraction Concept Test of sixty-eight items, constructed to measure the students' attainment of the sixteen selected subconcepts, was administered to 279 students in grades 4, 5, and 6. A student had to respond correctly to at least 75% of the items on a subtest in order to be described as attaining criterion on that subtest. The data were analyzed by using two ratios adapted from Gagné (1962) and Walbesser (1968) to determine sequencing validity.

Results

The results of testing the hypothesis were that eighteen of the twenty-three dependencies in the hypothesized hierarchy were supported, four were not supported, and no decision was possible for one dependency due to insufficient data.

Conclusions

The conclusion based on these results was that certain stated subconcepts were prerequisite to others. Some of these are:

1. Associating fractions with part-whole and part-group models were prerequisite to associating a fraction with a point on a number line.
2. Associating a fraction with a part-whole model or with a part-group model was prerequisite to using a fraction in a comparison situation involving the respective model.
3. Associating a fraction with a part-whole model or with a part-group model was prerequisite to associating a fraction with the respective model where the number of parts was a multiple of the denominator and the parts were arranged in an array that suggested the denominator.
4. Associating a fraction with a part-whole model or with a part-group model with congruent parts was prerequisite to associating a fraction with the respective model with non-congruent parts where, in the case of part-whole models, the parts were equal in area.

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THE RELATIONSHIP BETWEEN ADOLESCENT LOGICAL REASONING ABILITY AND NATURAL AND MODELED UTILIZATION OF THE LANGUAGE OF LOGIC

John W. Gregory
University of Florida

Purpose

This investigation was an outgrowth of several studies in a program of research at the Institute for Development of Human Resources and the Department of Secondary Education at the University of Florida. A major thrust of this program is to investigate teaching effectiveness in terms of teacher and student classroom verbal behaviors that are associated with student growth and development.

One teacher behavior which has been related empirically with student growth in conditional reasoning ability is language of conditional logic (Gregory 1972, Gregory and Casteel 1974). It has been hypothesized that students may acquire the formal language structure needed for reasoning logically by modeling (Bandura 1963, Gregory 1972). The investigation to be reported sought relationships between student conditional reasoning ability and student utilization of the language of conditional logic, both natural and modeled, for each of grades 5, 6, and 7. A measure of effects on student interview performance due to the administration of the logical reasoning test served as an experimen[illegible] variable. This variable was seen as one which could establish mind-se[illegible] the student for further performance, or serving as a model for language behavior, or both.

Procedure

One hundred five randomly selected fifth, sixth, and seventh grade subjects were individually interviewed. The sample had been randomly assigned to two interview groups such that half were interviewed prior to the administration of the Cornell Conditional Reasoning Test and half were interviewed following the test administration.

The interview format was designed to obtain samples of adolescent verbal behavior in which logical relationships could be expressed. The four tasks included free response items, an analysis of logical relationships inherent in a picture, rule identification, and retelling a story which had been presented with a high frequency of conditional moves. The fourth task also yielded a measure of student ability to recall relevant information from an oral presentation. The conditional moves uttered by the subject in the first three tasks measured his normal utilization of the move while the fourth task served to supply a measure of his modeling tendencies. The data from the 2-by-3 design were submitted to analyses of variance and correlation using an apriori alpha risk of .05.

Results

As expected, a significant difference between fifth and seventh grade conditional reasoning ability was found. A significant interaction between grade and interview schedule resulted for the variables of modeling (Task 4), total frequency of conditional moves, and number of elements recalled from the oral presentation. The results of subsequent analyses of simple effects involved in the interaction will be reported.

With the occurrance of significant interaction effects, correlation coefficients were computed for the grade-interview group combinations. A significant correlation between conditional reasoning ability and frequency of conditional moves of both natural and modeled types occurred for seventh grade subjects who were interviewed after the test administration only. Other significant relationships found and author interpretations will be reported.

Conclusions

Results will be incorporated in procedural and interpretive decisions for future studies in the Florida program. For example, designs will now be used which will lead to the interactive effects of testing and instructional verbal strategies on student outcomes. The results stress effects due to testing in addition to replicating the work of others concerned with the developmental nature of logical reasoning ability. In future studies, this effect can be dealt with in a positive fashion if considered as an instructional variable or as one which establishes set for further instruction and subsequent learning.

The results also indicate that the common assumption that logical reasoning ability and language of adolescents are related may be in need of qualification. This investigation adds credence to hypotheses of others which suggest that adolescent logical abilities (test-wise as well as linguistically speaking) are more egocentric than truly hypothetico-deductive in nature.

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THE EFFECT OF TRAINING FOURTH GRADE, SEVENTH GRADE AND HIGH SCHOOL PUPILS TO PERFORM COLUMN ADDITION DIRECTLY OR USING TENS

Grayson H. Wheatley
Purdue University

Daniel O. McHugh
Seeger High School
West Lebanon, Indiana

Purpose

A survey of texts for teaching mathematics in the elementary school identified two methods being recommended for performing column addition. One method, here called Direct, would have pupils add from top to bottom or bottom to top adding each addend to the partial sum. A second method, here called Tens, has students look for addends which add to ten and by application of the commutative and associative properties find the sum adding those numbers first. There is broad disagreement among these authors on which method is best with conflicting justifications. While Marks, Purdy, and Kinney (1965), Swenson (1972), and D'Augustine (1968) recommend the Direct method, Riedesel (1967), Collier and Lerch (1969), Schminke, Maertens, and Arnold (1973), and Buffie, Welch, and Paige (1968) suggest the Tens method be taught. In a recent training study (Wheatley, 1974) it was found that fourth grade children were more efficient in column addition when they added directly rather than looking for addends which add to ten. Students trained to add directly were 15-17% faster than students trained to add by tens. Not only did the direct group have more attempted and more correct items than the tens group but students performed much better as a result of direct training either before or after training on tens. This study attempted to extend the previous investigation to older students.

This study attempted to compare two methods of column addition, Direct and Tens, for speed and accuracy. The performance of three ability levels as well as three age levels was examined. The following questions were asked: (1) Are students faster and/or more accurate using the Direct method than the Tens method? (2) Does the same pattern of performance hold for differing ability and grade levels?

Procedure

The subjects for this study were 381 fourth-grade, seventh-grade and high school pupils drawn from an Indiana school located in a rural area. All students in each of 16 classes were included in the study. The high school students were studying either general math or algebra. These high school students were in classes taught by one of the investigators. In each class the students were partitioned into two groups using the initial letter of their last name with A-L determining one group and M-Z another. A pretest consisting of 40 single-digit column addition problems with seven addends each was administered to all subjects. A table of random numbers was used to generate the addends so that no bias would be introduced favoring one method. A similar procedure was employed in constructing the two posttests. Following the pretest, each group received four days of training on one of the two methods with a posttest on the fifth day (Friday). The treatments were then reversed with those students trained on the Direct method receiving training on Tens and those subjects previously receiving training on Tens being trained on the Direct method.

Results

The effect of method was examined in two ways. First t tests were used to determine change in means from pretest to posttest I for each treatment group, and change in means from posttest I to posttest II. While both treatment groups showed significant improvement from the pretest to posttest I, (.001) the increase in mean number correct was much greater for the Direct group ($t = 11.04$ vs. $t = 5.83$). The results for number attempted paralleled this. Posttest II and Posttest I comparisons showed that the group which had improved so much under Direct training showed a decrement in mean number correct on posttest II after Tens training while the group receiving the Direct training following Tens training showed marked improvement. The second method of determining method effect was to compare the means of the two treatment groups on posttest I. Results on this were obtained from a 2 (Method) X 3 (Ability) X 3 (Grade) analysis of variance. The main effect for method was significant ($p < .01$) with the Direct group having the higher mean. Again the results for number attempted paralleled the number correct results. The other main effects of Ability and Grade were also significant but the pattern was the same as on the pretest. Analysis of interactions indicated that the Direct method was superior for each Ability and Grade.

The accuracy of the two treatment groups was compared by using the percentage correct of those attempted. There were no significant differerces in accuracy of the tests.

Conclusions

The results of this study demonstrate rather conclusively that adding a column of numbers directly is faster than adding by a Tens method. This was found to be true for different ability and grade levels. The caution of some authors against using the Tens method because more mistakes will be made seems unfounded; there was no difference in accuracy between the two methods.

In attempting to determine why the Direct method was more efficient, a task analysis was performed for each method. It was concluded that students are slower adding by Tens because they spend time studying the column and deciding how to proceed. Decisions must be made on whether any combinations which add to ten exist and if they do, which ones to use and in what order. Students also feel the need to check whether the problem was worked correctly. If no combinations adding to ten are found the time spent in analysis is entirely wasted since the problem must then be added directly. The student adding directly can begin to add immediately in every case since no decisions need to be made.

Friday
4:00-5:00 p.m.   RESEARCH REPORTING SECTION: NO. 286   3C

Presider: GARTH E. RUNION, Illinois State University
Normal, Illinois

Reports: EFFECTS OF SUBSTITUTING VECTOR PROOFS IN PLACE OF SOME TRADITIONAL PROOFS OF THEOREMS IN PLANE GEOMETRY

SISTER FELICIA TANZELLA, Salesian Sisters of St. John Bosco
Paterson, New Jersey

SYMBOLIC MODE PREFERENCE IN LEARNING MATHEMATICAL STRUCTURE

GERALD D. BRAZIER, Virginia Polytechnic Institute and State University
Blacksburg, Virginia

MATHEMATICS CONTEST HONOR ROLL STUDENTS

HENRY M. COX, Executive Director of the Annual High School Mathematics Examination
Lincoln, Nebraska

CORRELATES OF HIGH QUANTITATIVE VS. HIGH VERBAL APTITUDE IN TERMS OF BIOGRAPHICAL AND PERSONALITY DATA

P. DOUGLAS KINDSCHI, Sangamon State University
Springfield, Illinois

EFFECT OF SUBSTITUTING VECTOR PROOFS IN PLACE OF SOME TRADITIONAL PROOFS OF THEOREMS IN PLANE GEOMETRY

Sister Felicia Tanzella
Salesian Sisters of Saint John Bosco

Purpose

The major purpose of this study was to compare the effectiveness of substituting vector proofs in place of some traditional proofs of theorems in plane geometry.

Procedure

Twelve groups of students who were studying plane geometry were selected from three high schools--four from a public high school for boys, four from a public high school for girls, and four from a coeducational private high school, all within the same urban center, where the socioeconomic status is mostly low.

The schools were not asked to use any particular geometry books, but to use whichever ones were originally intended for use. Four different texts were used.

Six teachers, two from each high school, volunteered to carry out the investigation experiment. Each teacher had two groups--a control group and an experimental group. In the experimental group, the teacher omitted certain exercises, theorems, and examples from the same textbook that the control group used, substituting for these parts the Student's Manual compiled by the writer of this paper, and the vector proofs of the thirty theorems. The control group followed the textbook without any changes.

One hundred fifty copies of Vector Proofs for thirty theorems were given to the six experimental classes. They were also given 150 copies of the Student's Manual. The teachers were free to present the whole Student's Manual first, or as the need arose for each theorem.

Three hundred students took a Geometry Prognosis Test in September, 1972. In January, 1973 a Midyear Geometry Test was given. The final test, the Howell Geometry Test was administered in May, 1973.

Analysis

The raw scores of the Geometry Prognosis Test were converted to standard T-scores. An analysis of variance of the Prognosis scores indicated significant intergroup differences. Therefore, an analysis of covariance was made of the Midyear scores and End of the Year scores, using the Prognosis scores as the covariate.

The basic null hypotheses of the study were:

When a class is taught plane geometry completely from a traditional textbook, or when a class is taught from a Student's Manual on Vectors and thirty vector proofs of theorems, omitting those parts of the textbook most directly related to these proofs:

A. There is no significant difference, at the .05 level, between the mean standard score of the Geometry Prognosis Test and the mean standard score of the Midyear Geometry Test;

B. There is no significant difference, at the .05 level, between the mean standard score of the Geometry Prognosis Test and the mean standard score of the Howell Geometry Test at the end of the school year.

Results

Use of an analysis of covariance technique indicated that neither Hypothesis A nor Hypothesis B could be rejected at the five per cent level of significance.

The main factor in predicting the Midyear and the End of the Year scores was the Prognosis score.

The following showed significant F ratios for the Prognosis-Midyear and the Prognosis-End of the Year analysis:

Schools
Covariate 1

The results of the F ratios on programs done on the separate schools showed the following to be significant at the five per cent level:

Prognosis-Midyear analysis
    School 1, Covariate 1
    School 2, Groups
              Covariate 1
    School 3, Covariate 1

Prognosis-End of the Year analysis
    School 1, TXG
              Covariate 1
    School 3, Covariate 1

In school 1 there was a teacher effect--one teaching more effectively the control group, one teaching more effectively the experimental group.

The difference between the mean Midyear T-scores and the mean Prognosis T-scores was 4.5 points lower for the total control group than for the total experimental group.

The difference between the mean End of the Year T-scores and the mean Prognosis T-scores was 5.8 points lower for the total control group than for the total experimental group.

In both cases, the total experimental group showed less change.

The T-scores of the experimental groups were not statistically higher than the T-scores of the control groups, nor statistically lower. The experimental method did not keep the students from progressing.

Conclusions

The writer feels that the study of vectors as introduced in the Student's Manual of this paper can be given profitably to high school sophomores. She feels, however, that the study of geometric proofs with vectors would perhaps be more profitable for high school seniors. In practice, it seems the students need more exposure to vectors, before being able to prove geometric theorems by vectors.

An investigation might be made in the use of the vector method in other areas of mathematics where vectors are applicable, such as algebra and trigonometry. This investigation may be of value in determining the place of vectors in high school mathematics, and in which year of high school, vector proofs of geometric theorems may be most profitable.

SYMBOLIC MODE PREFERENCE IN LEARNING MATHEMATICAL STRUCTURE

Gerald D. Brazier
Virginia Polytechnic Institute and State University

Purpose

The purpose of the experiment was to investigate the role that symbolic mode preference plays in learning mathematical structures. Using the mathematical group paradigm introduced by Dienes and Jeeves in Thinking in Structures (1965), the study sought specifically to find the influence of algebraic and geometric modes of symbolic representation on both the rate of learning and the strategies employed during learning by the subjects.

Jacques Hadamard (1954) when investigating imagery in mathematical creativity was told by some mathematicians that, "they think in a purely algebraic way, with the help of signs; others always need a 'figurated representation' . . . (p. 86)." Such preferences for either an algebraic or a geometric representation of a mathematical idea reflect the classic dichotomy of mathematics into algebra and geometry--the abstract and the concrete. A dichotomy like this, if it truly exists among people working with mathematics as well as in the mathematics itself, is very important for those concerned with mathematics education. If there are differences in the way people think about mathematics then these differences will be reflected in the way they learn about mathematics.

Learning mathematics is learning structure - this is an interpretation strongly emphasized in the curricular reform movement (SMSG, UICSM, SSMCIS, etc.); the computational aspects of mathematics have been deemphasized in favor of concentration on overall structural relationships. Investigations of the learning of structure have arisen both from educators and from psychologists concerned with more complex aspects of thinking than the classic aspects investigated in concept formation research. Following the lead of Bartlett (1958), Dienes and Jeeves reported in a 1965 monograph, Thinking in Structures, on investigations into the processes of learning complex structures. In their results, Dienes and Jeeves reported finding regularities in the patterns of performance of their subjects. These regular patterns of performance in the learning of complex structures were characterized as strategies by Dienes and Jeeves in a fashion reminiscent of Bruner, Goodnow and Austin's focusers and scanners (1956). The discovery of strategies in learning structures has ignited interest that has been expressed in several recent studies (Branca, 1971; Chilewski, 1973; Kellog, 1973).

As indicated above, the existence of preferences for different modes of symbolic representation of mathematical ideas has been suspected and attested to for a long time. Some people think algebraically; others think geometrically--so goes the folklore. Hadamard's interviews, reported in 1954, are not scientific evidence yet have a great amount of credence among mathematicians and educators. In the literature of experimental psychology, symbolic mode preference has been considered part of the more general question of stimulus preference. Underwood (1966) points out that certain stimulus biases must to a certain extent determine stimulus selection and so in a simple concept attainment task have a great effect on identification of the concept. The same biases would continue to have an effect in a more complex task such as learning the structure of a mathematical group. Underwood continues to say that he has little evidence of investigations of stimulus selection habits as a function of an abstract-concrete dimension. It was precisely this dimension that was investigated by determining subjects' preferences for algebraic or geometric modes of presentation.

Procedure

Each of 47 college students was given two experimental tasks in a single session of approximately an hour. In each task the subject and the experimenter had a set of cards, each corresponding to the elements of a mathematical group. The play of cards, one by the experimenter and one by the subject, corresponded to combining two elements of the group. The goal was to learn to correctly predict the results of these combinations and so learn the structure of the group. The experimenter kept a record of the subject's plays.

The first task was based on the Klein-four group. In this task both algebraic and geometric symbolic modes were present on each card and the subject was free to choose either mode in which to function. After completing this task, the subject was asked to pick from another set of cards four cards that could be used in playing the game. By his choices, the subject revealed which symbolic mode he preferred, algebraic or geometric.

The second task was similar in format to the first but was based on the Cyclic-four game. In this task only one symbolic mode was present for each subject, either algebraic or geometric. Half of each preference group had their preference matched and half did not. This partitioning gave rise to a 2 X 2 factorial design with these four cells: AA - algebraic preference with algebraic task mode, AG - algebraic preference with geometric task mode, GA - geometric preference with algebraic task mode, and GG - geometric preference with geometric task mode.

Three criterion variables were used. The first was the number of trials needed to understand the structure--understanding being defined as the ability to correctly predict the results of combining the cards. The second two variables were operator and pattern strategy scores as defined by Dienes and Jeeves. A multivariate analysis of variance was conducted to test for both main effects, preference and task mode, and for interaction.

Results

The results showed a significantly large ($p < .05$) multivariate F for both main effects. The interaction of preference and task mode was not significant. The univariate tests on the preference effect showed significance for the two strategy variables, operator ($p < .05$) and pattern ($p < .01$), but not for the rate of attainment variable. For the task mode effect, the univariate tests showed only significance for the pattern variable ($p < .01$). A plot of cell means showed that subjects with a geometric preference had a higher operator score and those with an algebraic preference had a higher pattern score. This interaction between strategy variables and preference was further analyzed by several χ^2 tests. These tests revealed that the mismatched groups, AG and GA were the source of the significant results.

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MATHEMATICS CONTEST HONOR ROLL STUDENTS

Henry M. Cox
Executive Director of the Annual
High School Mathematics Examination

Purpose

The Twenty-Fifth Annual High School Mathematics Examination sponsored jointly by the Mathematical Association of America, The Society of Actuaries, Mu Alpha Theta, The National Council of Teachers of Mathematics, and The Casualty Actuarial Society was held on Tuesday, March 12, 1974. More than 340,000 examinations were distributed to about 7,000 high schools in the United States, Canada, and the United Kingdom. The examination proved to be one of the "easier" ones; other "easy" examinations were those given in 1968 and 1970. As a consequence of the easier examination, the number of contestants with scores of 80 or above out of a possible 150 was larger than in any previous contest; there were 850 names on the Individual Honor Roll printed in the SUMMARY OF RESULTS AND AWARDS for 1974, pages 38-57.

Procedure

Questionnaires were sent to all schools having honor roll contestants. Statistical and anecdotal analyses have been made of the 685 responses received from schools in the continental United States.

Results

Fifty-five percent of the honor roll contestants had participated in one of more prior year contests, sixty percent were high school seniors, and twenty-five percent were juniors. There were four perfect scores, all boys, all of whom had scores of 800 on mathematical ability and scores of from 640 to 800 on verbal ability as measured by the Scholastic Aptitude Test of the College Entrance Examination Board.

It is to be remarked that nine boys on the honor roll are identified by their schools as freshmen, their ages 13, 14, or 15. All are reported as having mathematical sophistication. In addition, we find these comments: most valuable player on football team; an expert violin player; perfect score on Coop Geometry Test; calculus and chemistry; currently in analytical geometry; accelerated to fourth year mathematics in grade 9 on basis of special testing; studied algebra 1 and 2 independently and took geometry in eighth grade and trigonometry and analytic geometry in ninth grade, fortunately we are in a metropolitan area with major universities - we have a Scholars Program for high ability students, but special provisions still must be made for the exceptionally gifted student in mathematics. Some fifteen year old boys are high school seniors, not freshmen. For example the report on one student is that he participated in the William Lowell Putnam Mathematical Competition with a score of 39, rank 116. His scores on the CEEB Scholastic Aptitude Test were 600 Verbal and 800 Mathematical. He has completed advanced calculus at a local college. Incidentally he has taken the City Championship as a swimmer.

The number of girls included on the honor roll is small, however there were only ten boys with scores higher than the highest scoring girl. She was a senior. Her scores on the Scholastic Aptitude Test of the College Entrance Examination Board were 790 Verbal and 800 Mathematical; her scores on the CEEB Achievement Tests were 800 Mathematics, 800 Chemistry, 800 Physics, 800 English, and 800 French. Her teacher writes that this student spends a great deal of time outside of school reading mathematical literature, that she tutors other students in mathematics, and that she was a participant in the Third U.S.A. Olympiad. Eight other girls made scores which qualified them to participate in the Third U.S.A. Olympiad. Comments were: National Merit winner, worked over a year on independent study in genetics, active in music; National Merit finalist, Junior National Honor Society; has taken Algebra I and II, Geometry, and senior mathematics; tied for fifth place in Chicago area on National French Contest, took eighth grade Algebra by independent study; WSB Young American, National Merit Scholar, Key Club Leader, first in State Spanish Test, Circulo Hispanico Award; Valedictorian, All Conference Basketball, National Honor Society, Iowa State Scholarship Award in Mathematics; Gifted Students Program, National Merit finalist, first prize in UCSD Honors Mathematics Contest (1973).

CORRELATES OF HIGH QUANTITATIVE VS. HIGH VERBAL APTITUDE IN TERMS OF BIOGRAPHICAL AND PERSONALITY DATE

P. Douglas Kindschi
Sangamon State University

Purpose

Why do some people demonstrate high quantitative aptitude and others high verbal aptitude? This is the basic question which motivates the study. Following a review of the literature, the study defines selective superiority and outlines the research design which studies environmental biographical and personality correlates of differential aptitudes by use of a questionnaire. The results and conclusions are summarized.

In reviewing the literature, the most obvious correlate of differential aptitude is that which involves gender. It is well established that males tend to score higher on quantitative tests and that females perform better on verbal exams. Patterns in course electives and college majors show a larger percentage of males choosing mathematics and other quantitative related fields. Studies tracing differential aptitude factors in high school, elementary school, and even pre-school family environment are reported. Absence or presence of father in pre-school years has shown to be a significant factor. Sex role identification is correlated with problem solving ability and the sex related differences have been traced to one of the first learning experiences in infants, namely, that of mother identification.

Other studies relate the sex variable to differential aptitude by examining various national educational practices, autobiographies of famous people and pre-school environment. Other relationships with differential aptitude include physiological measurements, birth order, size of town, and such personality characteristics as desire for security, religious attitudes and measures of compulsiveness.

Procedure

The empirical study begins with a definition of selective superiority as demonstrated superiority in one measure of aptitude accompanied by inferior performance on the other measure among college freshmen. The two aptitudes measured are the quantitative and verbal aptitudes as tested by the College Qualifying Testing. Verbal selective superiority and quantitative selective superiority are thus defined. In order to avoid confusion of selective superiority with sex differences, the study is divided by gender and the male verbal selectively superior student and likewise for the female students. The questionnaire seeks information in areas such as: motivation, self-confidence, work style, attitudes and people orientation. Results of the questionnaire are analyzed by calculating chi-square and exact probabilities.

Results

The results (at .05 probability level) show significant differences between the high verbal and high quantitative students in such areas as: size of town during the pre-school years, number of school changes, early reading patterns, interest in political activity, compulsiveness, birth order, study patterns, social attitudes, and athletic ability. The relationship between these findings and the teaching of mathematics is discussed.

Saturday
8:30-9:30 a.m.   RESEARCH REPORTING SECTION: NO. 312   3C

Presider: JOSEPH N. PAYNE, University of Michigan
Ann Arbor, Michigan

Reports: CONCEPT ATTAINMENT PROBLEMS IN TWO SETTINGS

LARRY SOWDER, Northern Illinois University
DeKalb, Illinois

STRUCTURAL AND LINGUISTIC VARIABLES IN VERBAL SIMPLE DEDUCTIVE ARGUMENTS

JERRY R. SHIPMAN, Alabama A.&M. University
Normal, Alabama

A SEARCH FOR READING DIFFICULTIES AMONG ERRED WORD PROBLEMS

J. D. KNIFONG, West Virginia University
Morgantown, West Virginia

BOYD D. HOLTAN, West Virginia University
Morgantown, West Virginia

ORAL READING AND VOCABULARY LEVEL AS FACTORS IN ARITHMETIC WORD PROBLEM ACHIEVEMENT

MELVIN A. MCGILL, David O. Duncan Elementary School
Gary, Indiana

C. A. WILES, Indiana University Northwest
Gary, Indiana

CONCEPT ATTAINMENT PROBLEMS IN TWO SETTINGS

Larry Sowder
Northern Illinois University

Purpose

1. To compare performance on conjunctive and disjunctive concept attainment problems in two settings: geometric and "meaningful."

2. To compare transfer effects in moving from the geometric setting to the "meaningful" setting (and vice versa) on performance on such problems.

The wealth of work in concept attainment studies has dealt largely with arbitrarily defined concepts over contrived domains--e.g., the concept "two red triangles" over a domain made up of cards with numbers of colored geometric shapes drawn on them. Few investigations seem to have compared performance on such artificial tasks with performance on logically similar but recognizably more meaningful tasks--e.g., determining from past cases which symptoms might enable one to diagnose a particular disease. In addition, there seems to have been little examination of transfer effects from, say, an artificial setting to a more meaningful one.

In the same way, research on problem solving may be handicapped by the lack of problem material which is completely new to the Ss. One procedure is to use novel problems and hope that the Ss have not been exposed to similar problems or fruitful heuristics in some unmeasurable way. An alternative is to use problems which are in artificial settings but which are therefore almost certainly novel to the Ss. Whether performance on artificial problems is positively related to performance on more realistic problems cannot be assumed, but nonetheless seems to be uninvestigated. There would be little point in continuing to work with artificial tasks if their artificiality results in a performance which is unrelated to performance in more nearly "real life" situations.

Procedure

The problems used in the study were concept attainment problems. Booklets presenting conjunctive (...and...) and disjunctive (either...or... or both) concept attainment problems in a geometric and a "meaningful" setting were prepared. Concepts were two-valued and based on 3 x 3 x 3 x 3 domains. Characteristics used in the geometric domain were shape (triangular, square, circular), number of shapes (1, 2, 3), shading (dotted, lined, solid), and number of checks (1, 2, 3). Characteristics used in the "meaningful" setting were location of headache (front, top, back), blood pressure (too low, okay, too high), rash (slight, moderate, extensive), and temperature (below normal, normal, above normal). Problems in the two settings were matched in logical difficulty and, to an extent, domain attributes of the same relative saliency were matched.

Ss were college students in pre-service mathematics or mathematics methods courses (n = 90). Each S received only conjunctive or disjunctive tasks, with the first part of the booklet explaining the appropriate type. Booklets were randomized and distributed to Ss. Successive pages of the booklets identified examples/non-examples of each concept, until 6 such cases were presented. All cases encountered on previous pages and the description of the domain were repeated on subsequent pages to lessen Ss' memory strain. For example, the third page of a disjunctive concept problem might contain the cases given in figure 1. Four cases were necessary and sufficient to identify the concept.

Ss were to offer their best guesses as to the two defining characteristics for the concepts after each additional case. After three problems in one of the settings, Ss worked three more problems in the other setting. Performance was measured by the number of concepts correctly identified. Patterns of responses to examples/non-examples were also examined.

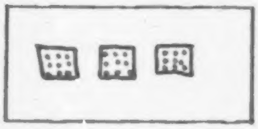
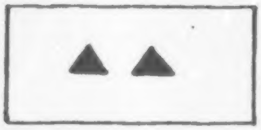
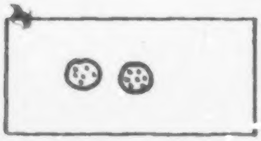
(Geometric domain)		("Meaningful" domain)	
	is an example.	top-of-head headache too high blood pressure slight rash normal temperature	came down with the disease
	is an example.	front-of-head headache okay blood pressure extensive rash normal temperature	came down with the disease
	is not an example.	back-of-head headache okay blood pressure slight rash below normal temp.	did not come down with the disease

Fig. 1 Cases on third pages of sample disjunctive concept problem.

Analysis

The proportions of concepts correctly identified were compared by using the arcsine transformation of the proportions and analyzing these transformed scores by a chi-square approximation (Hald, A., Statistical Theory with Engineering Applications, 1952, p. 712). Performance on the first three tasks was considered for purpose (1); performance on the last three tasks, for purpose (2).

Results

1. As in other studies, conjunctive concepts were more easily attained than disjunctive ones. Ss experienced greater difficulty in responding to non-examples in a manner consistent with all the information available to them.

2. The following table summarizes the transfer findings:

	Proportion Correct	
	Ss with conjunctive tasks	Ss with disjunctive tasks
First meaningful, then geometric	.75	.27
First geometric, then meaningful	.92	.60

The analysis indicated rejection of the equal-proportions hypothesis at the .005 level. Note the relatively poor performance on geometric disjunctive tasks after working with meaningful disjunctive tasks.

Conclusions

Insofar as the results are generalizable, this study indicates that,

1. disjunctive concept problems are more difficult than conjunctive concept tasks, regardless of the artificiality/meaningfulness of of the setting, and

2. for concept attainment problems, transfer effects from one setting to another depend on the type of concept and the difference in artificiality of the settings. Whether tentative conclusion (2) also holds if cases from both domains are presented verbally, rather than one verbally and the other pictorially, as in this study, is currently being investigated.

STRUCTURAL AND LINGUISTIC VARIABLES IN VERBAL SIMPLE DEDUCTIVE ARGUMENTS

Jerry R. Shipman
Alabama A. & M. University

Purpose

A number of recent studies by Jerman and others (e.g., Jerman and Rees, 1972; Jerman, 1973 a, b; Krushinski, 1973; Beardslee and Jerman, 1973, 1974; Cook, 1973, 1974) have focused on the influence of structural and linguistic variables on the relative difficulty of verbal arithmetic and algebra problems. Jansson (1974) conducted an exploratory investigation to identify those structural and linguistic variables which strongly influence the relative difficulty of subjects handling the judgment of simple deductive arguments in verbal form. The purpose of the present study was to refine and further investigate the variables explored in Jansson's study, identify new variables, determine which set of these variables would account for a significant amount of the observed variance in the error rate made by high school and college subjects on tests involving verbal simple deductive arguments, and attempt to shed light on a possible way curriculum developers and teachers can control the relative difficulty of such arguments.

A simple deductive argument in the present study refers to a chain of reasoning involving three propositions, where the first two propositions are premises (assumed to be true) and the third one is the conclusion drawn from the premises. If at least one of the premises is a conditional statement, then the argument is of the conditional type. Ennis and Paulus (1965), in the Cornell Critical Thinking Project, viewed the judgment of such argements as a significant component of the broader concept of critical thinking, which in turn was defined in terms of the notion of the "correct assessing of statements."

A number of other researchers (e.g., Hill, 1961; Roberge and Paulus, 1971; O'Brien, Shapiro, and Reali, 1971; Shipman, 1973; Tripp, 1973; Jansson, 1973) both within and outside the conceptual framework of the Cornell Critical Thinking Project, have studied the logical skills of subjects of a wide range of ages over such variables as type of logic reasoning (i.e., class and conditional), principle of reasoning (such as, modus ponens, contrapositive, converse, inverse, and transitivity), age, sex, content and context of arguments, negations, subject attitude and their interactions. In an overall sense, the variables investigated in these studies dealt with structural and linguistic variables (i.e., variables that are inherent in the structure of the problem itself) and learner characteristics.

Procedure

In the present study 18 structural and linguistic variables were used. Nine of the variables were used as defined in Jansson's study. The remaining 9 variables were either refined from Jansson's study or newly developed.

Two testing instruments, the Shipman-Tripp Conditional Reasoning Test, a 23-item test which was used in studies by Shipman (1973) and Tripp (1973), and the NiPalei-Rizza Class Reasoning Test, a 23-item test which was used in studies by NiPalei(1974) and Rizza (1974), were used to gather data. The items on the tests adhere to a common format and are designed to evaluate subjects' abilities to judge the validity of verbal simple arguments over four principles of deductive reasoning in two content dimensions.

The tests were administered to 10th, 11th, and 12th grade subjects and college freshman general mathematics students with little or no formal training in logical thinking.

The test results were subjected to the regression methods used by Jerman and his associates. Specifically, a linear regression analysis with a logarithmic transformation was applied to the test results, by using a stepwise regression computer program, to determine what specific structural and linguistic variables entered the model and in which order.

Results

In the case of the Shipman-Tripp test, the preliminary results after two sets of calculations involving the set of 18 structural and linguistic variables indicate that the order of entry of variables in the first three steps of the regression model is as follows: the principle of inference (PRINFR) variable entered in step 1, the validity status (VALIDY) variable entered in step 2, and the content type (CONTNT) variable entered in step 3. The regression coefficients, t-values, and correlations of all 18 variables entered into the model were calculated. The R^2 value at step 3 was approximately .69. For the NiPalei-Rizza test the PRINFR variable accounted for a major proportion of the variance in the difficulty, while the NEGPI variable increased the variance accounted for to 68%. The addition of the other variables did little to increase R^2. The regression coefficients with their t-values were calculated.

Conclusions

Based on these results, we are not yet in possession of satisfactory difficulty predictors, nor can we prescribe for the curriculum developer and teacher appropriate logic exercises (arguments) for secondary school and college students.

A SEARCH FOR READING DIFFICULTIES AMONG ERRED WORD PROBLEMS

J. D. Knifong
West Virginia University

Boyd D. Holtan
West Virginia University

Purpose

One of the most persistent assumptions about children's difficulties with word problems is that reading plays an important role. This assumption about the role of reading is so embedded in professional thinking that it remains in spite of rather severe challenges and weak support (Knifong and Holtan, 1975). The current study seeks to discover direct evidence of children's difficulties affecting word problem success.

The authors acknowledge the fundamental premise that *If a child cannot read a problem then he certainly cannot work it*. And they further acknowledge two following corrolaries: (a) *it is always possible to construct a word problem which can be solved by simple arithmetic operations and yet which is written in such a difficult style that grade school children cannot possibly decipher its meaning*; and (b) *a typical grade school classroom will contain a few students, usually one or two, who are dramatically behind grade level ability in reading and probably other subjects as well*. The issue is not whether reading is a *necessary* skill underlying success in word problems--that is assumed. Rather the issue is whether poor reading skills typically contribute to the average child's failure on common problems, or whether that failure is generally due to some other factor(s).

In a previous study Knifong and Holtan (1975) analyzed the written work of 35 sixth grade children on the word problem portion of the Metropolitan Achievement Test (1958) and the individual erred problems were placed into two disjoint categories. Category I contained all of those erred problems which could be positively identified from the child's written work as being the result of a clerical (3%) or computational error (49%). Category II contained all of the other erred problems, basically those problems which offered ambiguous or non-existent written evidence as to how and why they were missed. If there was any doubt about a problem or if there were apparently two sources of error, preference was given to Category II, thus tending two inflate the reported number of mistakes which *might* have been due to reading difficulties.

In about half of the erred problems (52%) the sole source of the error could be positively identified as computational or clerical in nature. Therefore, it was concluded that if reading difficulties contributed in a major way to these errors, they could not have been the cause for Category I erred problems and only might have been a contributing influence for

Category II erred problems. Hence there is now established evidence for recommending that teachers work on improving computational skills as a means of improving word problem success. In order to develop similar evidence, either for or against reading, each of the 35 students from the first study were interviewed 4-6 weeks after the written test. The interview included all problems which had been missed for any reason other than clerical or computational , i.e., problems in which poor reading skills might have been a contributing factor. (For a discussion of the sample population see Knifong and Holtan, 1975.)

Procedure

It was the intent of the investigators that each erred problem which was identified as belonging to Category II by either independent rater would be examined during the interview for possible reading impediments. Of the 240 erred problems which were to be included, 218 (91%) survived miscellaneous mishaps such as faulty recording device, mumbling, interviewer oversights, etc.

Each interview was conducted in the school during the school day and was recorded on tape. All interviews (except two) were completed in one setting. Since some of the children had to read as many as 20 problems and some as few as 2 or 3 problems the interview ranged from 10 minutes to 65 minutes in length. The reported data were compiled from audio recordings after the interviews were completed. Such a procedure establishes permanent data and serves as a control for interviewer bias since any misleading remarks or questions by the interviewer are recorded and can be taken into account as the rater interprets the child's response.

Each interview began with the reading of an introductory script which ended by asking the child to read problem 1. Since only one child missed this first problem in such a way that there was any doubt about his reading ability, it was used as a practice control problem for the interview. After this beginning, the missed problems assigned to Category II were considered in the order which they appeared on the test, and for each the interviewer would ask the child:

a. Please read this problem aloud.

After this first reading the interviewer would cover the problem and ask the following questions, pausing after each to allow a full, unhurried answer:

b. What kind of situation does the problem describe?

c. What does the problem ask you to find?

d. Do you know how to work this problem?

Although it was not part of the planned study, the interviewer would sometimes ask the additional question:

e. Would you look at the problem again and point out any words which gave you difficulty?

Since a person would normally re-read a word problem as he attempted to work it, in a similar manner the interviewer would uncover and allow the child to re-examine the problem at any point in the questioning. The problem was covered after the first reading primarily to encourage the child to use his own words in response to questions b and c, and not simply quote the relevant portions of the text.

Although a few children mentioned that they recognized the test as one which they had taken some weeks earlier, the interviewer gave no indication as to which problems had been missed and the child's previous work was not present or mentioned in any way. Nor was the child told about the purpose of the interview other than "I need you to read and discuss certain word problems so that I can tell how children think about word problems."

Since the audio recording was to provide a check on the inadvertent remarks which might bias a question, it was not considered necessary for the interviewer to read the script for each item and he was free to discuss or reword questions as the need arose. For example, question c was sometimes reworded for the child: "If you knew the answer to the problem, what would that tell you?" and for question e: "Are there any 'hard' words in that problem?" Such alternate wording is invaluable for coaxing a reluctant child to participate or to relieve anxieties which frequently occur when children first meet an adult stranger.

After all interviews had been completed two independent raters used the criteria described below to classify the various responses of the first four items (a, b, c and d).

Each of the major items covered by the interview were judged by independent raters as correct response, uncertain response or incorrect response. For each of the items the uncertain response was used when the response was judged ambiguous or those few times when it was non-existent. Since item e essentially asked the child to respond "yes" or "no" a second independent rating was not considered necessary for that item.

A. "Please Read This Problem Aloud."

In order for the auditory reading of a problem to be judged correct the problem had to be (1) read precisely by the child, (2) read with spontaneous corrections by the child, or (3) read with phrase or word substitutions which would not interfere with a third party being able to interpret and work the problem correctly. On the other hand, incorrect responses were characterized by number or word substitutions which would prevent a third party from interpreting and working the problem (e.g., reading "five" when a "g" appeared on the printed page).

Every child in the sample read aloud nearly all of the required problems without error. That is, occasional word substitution, mispronunciations and pauses (when the student encountered an unfamiliar word) were the only auditory reading errors found in this sample. Such occasional errors are a common feature of normal speech and do not necessarily reflect misunderstanding on the part of the child.

B. "What Kind of Situation Does the Problem Describe?" and
C. "What Does the Problem Ask You to Find?"

In order to be judged correct, the responses to items B and C had to describe the general context of the situation or the question being asked (respectively). Unlike auditory reading, if the child substituted a wrong number in his description of the situation (a relatively rare occurrence) it was not considered incorrect since these two items were seeking to assess the ability to interpret the written problem and that interpretation would be essentially the same for nearly all number substitutions. In addition, it is assumed that such errors are basically clerical in nature and would be corrected during the working of the problem, an activity which was not part of the interview procedure.

Incorrect responses were characterized by the child explaining a situation or question which was not presented in the problem. A child occasionally read the problem correctly and then described the context incorrectly. For example, after reading a problem about two girls bringing some sea shells for a science corner, one of the subjects described the context as "One girl brought sea shells and the other girl took some away." If the child offered a response and then spontaneously altered it, the last response was the one which the rater used in making his judgments.

D. "Do You Know How to Work this Problem?"

A correct response to this last item was characterized by the child indicating the operations and numbers to be used to solve the problem. It was not necessary for the child to name the numbers but only to indicate which ones would be used, e.g., referring to the sea shell problem, the child might say, "Add the two numbers together", or "Add all of the shells." Occasionally such directions became complicated in multi-step problems and children used phrases like, "And then you take that number and subtract the total amount paid." Incorrect responses were frequent and consisted of two main types: The child would shrug, smile and say, "I am not sure": or the child would indicate an incorrect operation or incorrect series of operations. As can be surmised from the data frequently children indicated a clear interpretation of the problem situation and the question being asked, but still did not indicate knowledge of how to work the problem.

Results

1. Response Frequencies

Table 1 about here.

The data on the first three items is so strongly against reading as a contributing factor that it is impossible to interpret the low reliability coefficient on the second item as a problem. Even if one were trying to assume an interpretation of the data favorable toward reading it would be difficult to imagine that factor accounting for any more than 10% of the erred problems. It should be noted that the relatively large number of instances where children did not know how to work the problem was not evidence that they did not know how to read and interpret the written problem, issues which are dealt with in the first three items.

Of those 126 times that the interviewer thought to ask the children if there were any words which gave them difficulty, only 10 responses were affirmative. At least 7 of these cases were due to a typographical anomally which seldom occurs in printed texts. (The work linoleum was hyphenated at the end of a line.)

2. Audio Reading Times

In a further attempt to define reading's elusive role, it was hypothesized that even though children could read and interpret the problems correctly, perhaps some might read so slowly as to interfere with success. Accordingly the verbal reading times for the first and last problems of the test were measured to the nearest half-second.

Insert Table 2 about here.

The computation of the correlations between reading time of the first problem or the last problem with (a) the number of problems left blank at the end of the test, (b) the total number of problems left blank, or (c) the total number of problems missed produced two interesting correlations above .50. (The total number of blank responses was counted for each child, as well as the number of blank responses at the end of the child's effort on the written test.)

Insert Table 3 about here.

In order to clarify the interaction between these factors, the students were grouped high and low (using the median of the measurements) and χ^2 was computed for: (a) high and low reading times of the last problem × high and low numbers left blank at the end of the test; and (b) high and low reading times of the last problem × high and low total number of problems left blank. The statistic was not significant at the .05 level in either case.

Insert Table 4 about here.

Finally, the students were rank ordered on the basis of reading time for the last problem, and the child who took the longest time to read the last problem also left the most total blank responses and the most blank responses at the end of the test. Further, the child who took the second longest time reading the last problem ranked third as leaving the most problems blank at the end of the test. After these two cases, the authors were unable to interpret further trends.

Insert Table 5 about here.

On the bases of the two correlations (.53 and .57) and the rank ordering of the students, it would seem that there is some evidence of slow reading acting as a deterrent to success. That is, the two extreme cases may be viewed as cases in which slow reading caused failure; however, there are three reservations concerning such a claim which should be mentioned. First, the fact that there were two significant correlations, neither of which produced a significant X^2, suggests that the principal interaction occurred with the extremes of the sample, a fact substantiated by the rank ordering of the students. Second, these two children may be having difficulty in both reading and arithmetic for some third undisclosed reason(s). In fact, Balow (1964) showed that the interaction between reading and word problems on a standardized test could be accounted for by the mutual interaction of these factors with IQ.

Third, consider the data in Table 2. The slowest reading rate measured (28.5 seconds for problem 30) was 82 words per minute and the average (for the last problem) was 154 words per minute. A person reading the 847 word test at these rates could finish in 10.3 and 5.5 minutes respectively. It seems hard to imagine that allowing the slower readers an extra 5 or 10 minutes on this extended 55 minute test would have improved their test score appreciably.

(Within the restraints of the testing procedure used in the first study it was not possible to determine whether slow reading habits would be a contributing factor for children taking a timed test. Nor was it felt that this question would be of principal interest to the practice of teaching.)

Conclusions

It was the primary intent of the investigators to discover direct evidence of poor reading abilities affecting children's success on word problems. We found little such evidence. On the basis of children's written work, the previous study excluded all of those problems worked correctly and all of these erred problems whose written responses provided direct evidence that the child had read the problem well enough to work it, but had committed either a clerical or computational mistake. This follow-up study interviewed the same group of children on those individual erred problems in which poor reading skills might have been a contributing factor.

The most obvious issue to be decided by the interview was, can the children read the problems aloud? Accordingly, each child was first asked to read aloud each problem which was suspect; the data indicates a high rate of success on this task. The most natural challenge to such success is whether the students understood what they were reading. In order to clarify this issue, the children were asked to explain in their own words the setting of the problem and what the problem was asking them to find. Again the data indicates that in the overwhelming majority of cases the children were successful in explaining both the situation and the questions of the problems which they had missed. Finally, the children were asked how they would proceed to work the problem and here the pattern of success is broken. In large numbers of cases, children (after responding correctly to the first three items) indicated that they did not know how to proceed to work the problem. But this can hardly be claimed as evidence of poor reading abilities.

Having failed to find direct evidence of reading difficulties by the methods above, one might suggest that the effect of reading is of a different nature, e.g., perhaps word problems are missed because some children are slow readers. Accordingly, the reading times of the first and last problems of the test were measured to the nearest half-second and correlations were computed between these items and (a) the total problems left blank, (b) the number of problems left blank at the end of the test, and (c) the total number of problems missed. The two strongest correlations are between the reading time of the last question and (a) the total number of problems left blank, and (b) the number of those left blank at the end of the test (.53 and .57 respectively). However, when was computed between high and low groupings on both of these factors, no significant difference was found. Last, students were rank ordered according to the time they took to read the last problem, and evidence emerged that the two slowest readers of the last problem also left a large number of problems blank. Hence, reading habits of two of the 35 students may have been so slow as to contribute to their errors. This evidence for reading is in spite of the fact that of the 1050 student/problems (35 students x 30 problems) fully 470 (45%) were missed.

It is always much easier to document the existence of a phenomenon (which in fact does exist) than to document the nonexistence of a unicorn. In this study there are a number of loopholes which prevent the assertion that reading could not be a contributing factor to children's failure. A partial listing of these issues are: (1) it is entirely possible, though not very probable, that reading does contribute to children's general difficulties with word problems in a manner not yet considered; (2) it may be that reading contributes significantly to word problem difficulties in fourth grade arithmetic or possibly ninth grade algebra, but not in the sixth grade; (3) it might have been that this particular group of West Virginia students were exceptional in some way which allowed them to miss a large number of word problems for reasons other than reading; or (4) one might want to consider the type of word problem used: Is a set of standardized test questions really representative of the universal set of all sixth grade word problems?

These and similar questions can always be raised concerning experimental studies, but they become particularly crucial if the nonexistence of a factor is being proposed. Hence the strongest claim which can be made from this data is that _if_ reading affects word problem success to a significant extent it is not revealed by these methods. However this fact, along with rather limited research evidence (Martin, 1963 and refutation Balow, 1964; Lyda and Duncan, 1967 and refutation, Kane, 1967; Knifong and Holtan, 1975, for a brief review), seems a good basis for challenging those who propose reading as an important factor.

Until someone accepts this challenge and does in fact provide positive evidence of reading ability affecting word problem performance, the best recommendation which can be made to the practicing teacher would be to help the class work on computational skills. And do not expect work on reading skills (which may be valuable in its own right) to correct difficulties with word problems except for those individuals who are dramatically behind grade level in reading ability.

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Table 1

The Mean Frequency of Responses to the Four Major Interview Items

Mean Frequency of Responses to the Directive/Question:	Correct Response	Uncertain Response	Incorrect Response	Total No. of Disagreements[a]
"Please read the following problem aloud."	208 95%	1.5 1%	8.5 4%	6
"What situation is the problem describing?"	214 98%	2 1%	2 1%	8
"What is the problem asking you to find?"	201.5 92%	6.5 3%	10 5%	20
"How would you work this problem?"	78 36%	37 17%	103 47%	43

Note: The criteria used by the rater to judge these responses are described in the text.

[a] The reliability, corrected by the Spearman-Brown formula is: for the entire table, .85; for the first item, .82; for the second, -.03; for the third, .57; and for the last, .81. The small coefficients for the second and third items is attributed to the small numbers in the Uncertain and Incorrect Response categories. E.g., the second rater only classified one problem out of the 218 as being uncertain and none as being incorrect.

Table 2

Reading Times for the First and Last Test Items

Problem 1.

Ruth brought 15 sea shells for our science corner and Betty brought 7. How many shells did the two girls bring?

N = 35	
Total number of words	21
Total number of sentences	2
Mean reading time	7.6 seconds
Range	6.0--11 seconds
Average reading rate	168 words per minute
Range	210--114.5 words per minute

Problem 30.

Carol is paid 80¢ an hour for babysitting. On Friday she worked from 8:30 to 10:30 in the evening and on Saturday from 7:00 to 9:45 in the evening. How much money did she make during the weekend?

N = 25*	
Total number of words	39
Total number of sentences	3
Mean reading time	15.2 seconds
Range	10.5--28.5 seconds
Average reading rate	154 words per minute
Range	223--82 words per minute

Note: The total number of words on the test was 847.

*Since the children were asked to read only those erred problems in which reading might have been a factor, only 25 of the 35 students were asked to read the last problem.

Table 3

Correlation Coefficients for Interview Reading Times X Test Problems Left Blank or Missed

	Total No. Missed	Total No. Left Blank	No. Left Blank At End of Test
Reading Time of First Question	.31	.48	.40

$N_1 = 35$

	Total No. Missed	Total No. Left Blank	No. Left Blank At End of Test
Reading Time of Last Question	.46	.53	.57

$N_2 = 25$

Table 4

χ^2 for High and Low Interview Reading Times x Problems Left Blank on Test

		Total Number of Problems Left Blank	
		Below Median	Median or Above
Reading time of last problem	Below Median	7	5
	Median or Above	5	8

$\chi^2 = .99$ Median of Reading Time = 14
N = 25 Median of Total Blanks = 4

		Number of Problems Left Blank at End of Test	
		Below Median	Median or Above
Reading time of last problem	Below Median	7	5
	Median or Above	6	7

$\chi^2 = .37$ Median of Reading Time = 14
N = 25 Median of End Blanks = 2

Note: .05 level of significance for χ^2 is 3.841

Table 5

A Rank Ordering of the Eight Longest Reading Times for the Last Problem

Time in Seconds	Total Missed	Total Left Blank	No. Left At End	Time to Read First Problem (seconds)
28.5	24	17*	15*	10
21.5	16	10	10	8.5
18.5	18	2	0	6.5
18	13	4	3	8
18	15	0	0	7.5
17	22	12	5	8.5
17	25*	16	5	11*
16.5	11	3	1	10.5

*Indicates the greatest number of problems in this category for the 35 students.

Saturday
11:30-12:30 p.m. RESEARCH REPORTING SECTION: NO. 351 3C

Presider: WALTER SZETELA, University of British Columbia
Vancouver, British Columbia, Canada

Reports: A COMPARISON STUDY OF THE EFFECTS OF THREE INSTRUCTIONAL STRATEGIES ON PROBLEM SOLVING BEHAVIORS

KENNETH E. VOS, College of St. Catherine
St. Paul, Minnesota

THE EFFECT OF VERBALIZATION OF HEURISTICS ON TRANSFER OF LEARNING

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Miami, Florida

A COMPONENT ANALYSIS OF MATHEMATICAL PROBLEM SOLVING

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Sterling, Kansas

GRAYSON H. WHEATLEY, Purdue University
Lafayette, Indiana

PROCESSES INVOLVED IN MATHEMATICAL PROBLEM SOLVING

MARY GRACE KANTOWSKI, University of Florida
Gainesville, Florida

A COMPARISON STUDY OF THE EFFECTS OF THREE INSTRUCTIONAL STRATEGIES ON PROBLEM SOLVING BEHAVIORS

Kenneth E. Vos
College of St. Catherine

Purpose

In mathematics there are selected learner behaviors which can be identified as important for solving mathematical modeling problems. Included in these behaviors are drawing a diagram, approximating and verifying, constructing an algebraic equation, classifying data and constructing a chart. The study compared three instructional strategies for promoting these behaviors. Each of the treatments involved working on a mathematical problem, but with variations in the placement of, and emphasis on, an implied behavior. The three experimental treatments, classified as R, L and B, were defined as follows:

R - (Repetition) - subject given only the problem task,
L - (List) - subject given the problem task, after a time lapse a checklist of desirable behaviors and specific written instruction in a behavior that could help in solving the problem task, then return to problem task,
B - (Behavior Instruction) - subject given specific instruction in a behavior, then the problem task.

Procedure

The experimental setting was within six mathematics classes across three grade levels (9, 10, 11) at Unity Christian High School in Orange City, Iowa. The research design was a one-factor randomized complete block with mathematics classes as a blocking variable. The classes involved, class size in parenthesis, were as follows: Algebra II (25); Geometry (29); Math Survey (21); Elementary Algebra (8); Algebra I, Section 1 (22); and Algebra I, Section 2 (28).

Pretest data were compiled from Sequential Tests of Educational Progress (STEP), forms 2A and 3A, Mathematics Part II. The method of administration of instruction was through self-directed materials developed by the investigator with provisions for teacher input. Over a fifteen (15) weeks period, twenty (20) problem tasks were given. Posttest data were compiled from the following measures: STEP, forms 2A and 3A, Mathematics Part I, investigator constructed Problem Solving Approach Test and Problem Solving Test. Analysis of the data involved descriptive statistics of means and standard deviations in addition to ANOVA using unweighted-means procedures and a Newman-Keuls method of pairwise means comparisons.

Results

There was no significant difference in mean performance among the three experimental treatment groups within each class for problem solving ability as measured by STEP 2A or 3A, Mathematics Part I.

On the Problem Solving Test for mean partial scores, the treatment differences were significant at $p < .20$ for three classes: Algebra I, Section 2; Elementary Algebra and Math Survey. In the Math Survey and Algebra I, Section 2 classes, the highest mean partial score was the R-treatment group.

The B-treatment group exhibited the highest proportion of occurrences of the five problem solving behaviors in three classes. In two low ability classes, the L-treatment group had the highest proportion of occurrences and the R-treatment group had the highest proportion in the Geometry class.

The B-treatment group exhibited the highest proportion of occurrences of other problem solving behaviors in three mathematics classes. In two classes the R-treatment group had the highest proportion of occurrences and the L-treatment group had the highest proportion in the Algebra I, section 2 class.

On the Problem Solving Approach Test Part I related to applying the five problem solving behaviors, the treatment differences were significant at $p < .10$ and $p < .05$ respectively for two classes: Algebra I, Section 1 and Math Survey. For the Algebra I, Section 1 class the L-treatment group mean was significantly ($p < .10$) higher than the other two treatment means. For the Math Survey class the B-treatment group mean was significantly ($p < .10$) higher than the other two treatment means.

On the Problem Solving Approach Test Part II related to other problem solving behaviors, the treatment differences were significant at $p < .20$ for two classes: Math Survey and Algebra II. The results indicated that the two experimental treatment groups involving instruction, B and L, were more likely to exhibit problem solving behaviors.

ORAL READING AND VOCABULARY LEVEL AS FACTORS IN ARITHMETIC WORD PROBLEM ACHIEVEMENT

Melvin A. McGill
David O. Duncan Elementary School

C. A. Wiles
Indiana University Northwest

Purpose

The purpose of the study was to determine whether or not students who have word problems read to them would achieve higher scores than students who read the same problems themselves.

Procedure

Eighty-nine fifth grade students from an urban school in Northwest Indiana were randomly assigned to four groups. Groups one and two were given word problems from the Iowa Test of Basic Skills. Group one read and completed the test items in the usual manner. Group two received the items by means of a tape recorder in addition to the printed test. Groups three and four were given items from a problem solving inventory test designed by Joseph Robertson. This test differed from the Iowa Test in that it contained a lower vocabulary level. Group three followed the same procedure as group one. Group four followed the same procedure as group two. All groups were given as much time as needed to complete the test.

Conclusions

The mean scores of groups 1 and 2 were not significantly different. However the mean score of group 4 was significantly greater than that of group 3.

The findings of this study indicated that when a word problem test has a high vocabulary level, there tends to be no significant difference in the scoring of those students who were read the test items and those students who read the items themselves. On the other hand, when a word problem test has a low vocabulary level, students who have problems read to them tend to make significantly higher mean scores than students who read the problems themselves.

The two tests differed in vocabulary level and syntax, and possibly in content. However the results indicate that reading ability in the sense of translating symbols to words, is not the only factor that may be affecting problem solving performance. If the assumption is made that the content of the two tests is similar, the factors of vocabulary and syntax used in presenting the problem may be significantly related to student performance.

THE EFFECT OF VERBALIZATION OF HEURISTICS ON TRANSFER OF LEARNING

C. Edwin McClintock
Florida International University

Purpose

The purpose of this study was to investigate the performance of Ss on transfer tasks following varied instruction and heuristics verbalization. The transfer tasks measure application and generalization of mathematical principles learned during instruction. Instruction and heuristics verbalization were the independent variables. Three levels of instructional modes (self-instructional reading, "live" teacher instruction, and combined reading and "live" instruction), three levels of instructional method (discovery, expository, and control), and two levels of heuristics verbalization (verbalization of heuristics, and no verbalization of heuristics) were employed. It was predicted that those Ss allowed to verbalize the employed heuristics would tend to perform better on the transfer tasks and that this advantage would be independent of the instructional variables.

Research findings indicate that general heuristics facilitate transfer and that Ss benefit from instruction that employs problem solving training. This research seeks to answer the questions of whether verbalization of heuristics is independent of instructional mode or method and whether transfer is facilitated by instructional closure brought about through individual verbalization of heuristics employed on a practice set of transfer tasks.

Research on problem solving and interest in problem solving heuristics draws much of its impetus from the work of George Polya. Transfer of learning, on the other hand, has long been an educationally important concept. Among research supporting the employment of problem solving heuristics as a means of facilitating transfer are the works of Wills and Wilson. Each focused on an aspect of problem solving heuristics as an independent variable. In both cases, specific heuristics were taught during instruction; in both cases, heuristics were said to facilitate transfer.

This study contrasts with those of Wills and Wilson in that it does not attempt to teach specific heuristics during instruction; rather, it requires a portion of the subjects to verbalize their own employed heuristics.

This study draws upon theoretical and research literature in its design of instruction around the precepts of Ausubel, Bruner and Gagné as well. The contrasting views of these psychologists provide theoretical bases for the development of the instructional methods, each being in its own way designed to enhance transfer.

Procedure

The Ss were 120 secondary school students from private schools in Miami, Florida. All Ss were enrolled in either high school geometry or Algebra II. Ss were randomly assigned to a treatment combination in a 3 x 3 x 2 factoral design. Training was provided through either self-instructional reading of prepared booklets, carefully controlled "live instruction", or equal time amounts of "live instruction" and self-instructional reading in that order. In each instructional mode, the same fixed set of mathematical principles was taught.

Following initial learning, practice was provided through one hour of individual effort on transfer tasks. These tasks were similar to the items of the dependent variable measure; both these tasks and the transfer test were constructed from the same item pool. Following the practice session, the heuristics verbalization group responded verbally to a "free description" to "loosely structured" audio recording session of heuristics verbalization. The non-verbalization group was involved in non-experimentally related activities.

Data was collected through a researcher designed transfer test. The items required either (a) application of learned principles, (b) analysis of learned mathematical principles, or (c) derivation of new principles that are partially related or unrelated to the learned mathematical principles.

Analysis and Results

Although the analysis and results of this research are not available at the present time, they should be completed during September, 197 .

A COMPONENT ANALYSIS OF MATHEMATICAL PROBLEM SOLVING

Ruben D. Schwieger
Sterling College

Grayson H. Wheatley
Purdue University

Purpose

A review of the literature on problem solving in mathematics indicates a lack of identification of what is involved in the act of problem solving. Dewey listed the steps in problem solving and Duncker later specified conditions for problem solving. Thorndike focused on trial and error in problem solving but did not attempt to identify and study the other types of thinking problem solving. Polya provided suggestions for improving problem solving and Wertheimer brought a Gestalt orientation to the task and emphasized modified trial and error thinking. Thus, rather global descriptions appear with broad categories named but the dearth of definitive knowledge of problem solving is evidence of the need for more precise analysis. Furthermore, difficulties in problem solving have been attributed to rather arbitrary designations of problem difficulty or to the lack of experience with types of problems. Studies have described the effectiveness of certain teaching devices and materials which seem to facilitate problem solving, but few writers have been more specific than to classify problems as algorithmic or non-algorithmic.

The purpose of this study was to develop a theoretical model for a more complete analysis of mathematical problem solving. This model was based on the identification and analysis of basic component abilities of mathematical problem solving.

Procedure

To identify the basic components, the authors first generated from the literature a list of general problem solving abilities. Following several revisions, eight components were identified as the basic thought processes in mathematical problem solving. The eight components are the ability to classify, deduce, estimate, generate patterns, hypothesize, translate, use trial and error, and verify. Each of the eight components was carefully defined and examples and nonexamples were listed. Furthermore, the interrelationships between the components were explored, resulting in a hierarchical ordering of them on the basis of prerequisite tasks. For example, before a person verifies he hypothesizes.

Four methods were chosen to test the validity of the model and the basic nature of the component abilities. 1) It was possible to define each component ability in terms of noncomplex general thinking abilities, to define them independently of one another, and to explain more complex abilities in terms of these eight. 2) Further suggestion of the validity of the model came from the task analyses of problems. 3) Experts were given a list of the eight component abilities with their definitions, descriptions, and examples and asked for their opinions and comments. Their comments also suggested that these eight components are necessary and sufficient to explain and characterize mathematical problem solving. 4) As part of the development and validation of the model, subjects ages 13-30 were interviewed. During the interview subjects were asked to solve several non-algorithmic problems and to vocalize their thoughts as much as possible. The interview sessions were tape recorded and later analyzed. The analysis of these protocols provided evidence for the exhaustiveness of the eight components; each thought pattern observed could be identified as an example of one of the components.

Conclusions

The model has immediate application in (1) analyzing the difficulty of problems, (2) designing instruction in terms of the components, (3) identifying the source of difficulty for unsuccessful problem solvers, and (4) as a paradigm for future research in mathematical problem solving.

The limitations of the study lie in the need for further validation of the model and the test of usefulness by teachers and researchers. Additional steps are needed to make the model effective in practice, e.g., instruments to assess each component ability.

Analysis of strategies used in problem solving could be conducted using the model as a frame of reference. The thinking involved in learning mathematics as opposed to application type problem solving could be similarly analyzed. Levels of functioning within each component may be identified and applications found to characterize the thinking in problem solving more precisely. As mentioned before, instruments need to be developed to ascertain whether an individual can use each of the components and whether he can integrate them.

PROCESSES INVOLVED IN MATHEMATICAL PROBLEM SOLVING

Mary G. Kantowski
University of Florida

Purpose

The ability to solve complex, nonroutine problems is one important goal of mathematics education. There are two essential components of the problem solving experience; the process, or set of behaviors that lead the problem solver to his goal, and the product, or actual solution. Most research in this country has dealt with the product aspect of problem solving. The dependent variable is typically some objective measure in which numbers of correct solutions are used to indicate success in problem solving. The purpose of this research was to uncover information about processes involved in successful problem solving. It was a clinical exploratory study to detect processes used by subjects as they learned to solve problems in geometry. The object of the study was not to state conclusions, but to seek regularities to suggest hypotheses for future experimental studies.

Newell and Simon postulate that problem solving consists in a search through a problem space. The rules for choosing the search paths through the space are heuristics or "rules of thumb" to aid the problem solver in finding the solution. Many heuristics, such as the means-end and planning heuristics, the use of looking back strategies, and working backwards, have been described by Polya. The search for a solution of a problem is executed by the program whose basic operators may be broadly classified as analysis and synthesis.

In the present study, the main processes of interest were the use of heuristics and the analytic-synthetic deductions exhibited by the problem solver. The sequence of the processes was also of interest.

Procedure

The study was comprised of four phases; a pretest, a readiness instruction phase, an instruction in geometry phase, and a posttest. During each phase, the subjects were asked to "think aloud" as they solved problems, and their protocols were recorded on cassette tapes and later analyzed according to a modification of the coding scheme developed by Kilpatrick in his dissertation research. A process-product score was assigned to each problem. This score, as the name indicates, was dependent on the processes used as well as on the solution. A median score was found for each of the eight subjects using his own distribution of process-product scores. Percentages of problems in which processes of interest were used were calculated for problems with scores above and below the median.

Results

Table 1

Percent of Solutions Indicating
Use of Goal-Oriented Heuristics

Subject	Problems with Scores Above the Median	Problems with Scores Below the Median
F-1	91	5
F-2	78	18
F-3	87	52
F-4	95	18
M-1	59	0
M-2	92	26
M-3	74	24
M-4	73	23

Table 1 indicates that 59 to 95 percent of the solutions with scores above the median showed evidence of the use of heuristics, while at most 52 percent of the solutions with scores below the median gave any indication of their use. Moreover, the use of heuristics was consistently more evident in scores above the median, that is, the percentage was higher for each subject for scores above the median than for scores below the median.

Furthermore, Table 2 indicates that the tendency to use heuristics increases as problem solving ability develops. From 17 to 50 percent of solutions in Test Block 1, median percentage 25, 32 to 84 percent of solutions in Test Block 2, median percentage 47, and 31 to 100 percent of solutions in Test Block 3, median percentage 77, showed evidence of the use of heuristics.[1]

Table 2

Percent of Solutions Indicating
Use of Goal-Oriented Heuristics

Subject	Test Block 1	Test Block 2	Test Block 3
F-1	17	47	77
F-2	33	47	69
F-3	42	84	77
F-4	25	58	77

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[1] Test Block 1 included test items administered before the instruction phase, Test Block 2 included items administered during the instruction phase, and Test Block 3 included the Posttest items.

Table 2 (continued)
Percent of Solutions Indicating
Use of Goal-Oriented Heuristics

Subject	Test Block 1	Test Block 2	Test Block 3
M-1	25	32	31
M-2	50	42	100
M-3	25	42	85
M-4	17	53	69

Table 3 presents percentages of the use of heuristics on corresponding pretest and posttest items. From 14 to 72 percent of the solutions on the pretest, with a median percentage of 36, and 14 to 100 percent of the solutions on the posttest with a median percentage of 72, exhibited evidence of the use of heuristics.

Table 3

Percent of Solutions Indicating
Use of Goal-Oriented Heuristics
on Corresponding Pretest and Posttest Items

Subject	Pretest	Posttest
F-1	14	72
F-2	43	72
F-3	57	72
F-4	29	72
M-1	29	14
M-2	72	100
M-3	43	86
M-4	29	72

Evidence of regular patterns of analysis and synthesis was noted in the solutions of problems with scores above the median. Analyses inferred from hypotheses were followed by syntheses, then by further inferences from the new synthesis and so forth. In a high percentage of cases these regular patterns of analysis and synthesis were immediately preceded by a goal-oriented heuristic. In cases where non-goal-oriented heuristics were introduced, the patterns of analysis and synthesis were generally irregular. The introduction of a heuristic related to the goal seemed to correspond to "insight." From that point, the path to the goal was, in most cases, clear, and characterized by regular analysis-synthesis patterns.

Table 4 below suggests that the manifestation of regular patterns of analysis and synthesis among successful problem solvers is striking. As the table indicates, from 77 to 100 percent, with a median percentage of 95, of the solutions with scores above the median disclosed these patterns, while only 18 to 41 percent of the protocols, median percentage 23, of the protocols from solutions with scores below the median demonstrated this pattern sequence. Thus, more regular patterns of analysis and synthesis are observed in successful problem solvers.

Table 4

Percent of Solutions Indicating
Regular Patterns of Analysis and Synthesis

Subject	Problems with Scores Above the Median	Problems with Scores Below the Median
F-1	95	23
F-2	95	18
F-3	96	38
F-4	95	41
M-1	77	23
M-2	100	21
M-3	96	24
M-4	100	23

That there is an interrelationship between the use of goal-oriented heuristics and the sequence of process patterns is evident from Table 5.

Table 5

Percent of Solutions Indicating
Use of Goal-Oriented Heuristics and
Regular Patterns of Analysis and Synthesis

Subject	Problems with Scores Above the Median	Problems with Scores Below the Median
F-1	86	5
F-2	73	9
F-3	78	33
F-4	91	14
M-1	50	0
M-2	92	11
M-3	74	0
M-4	73	9

From 50 to 92 percent of solutions with scores above the median showed indications of both regular patterns of analysis and synthesis and the use of heuristics, while at most 33 percent of the solutions with scores below the median indicated a combination of these variables.

Conclusions

As was stated initially, the purpose of this research was to suggest hypotheses for future studies. Results of this exploratory study indicate that the use of heuristics may be closely related to success in problem solving and should be investigated experimentally.